Title: Riemannian metrics having common geodesics with Berwald metrics

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In Theorem 1, we generalize some results of SZABÓ [**?**], [**?**] for Berwald metrics that are not necessarily strictly convex: we show that for every Berwald metric F there always exists a Riemannian metric affine equivalent to F. As an application we show (Corollary 3) that every Berwald projectively flat metric is a Minkowski metric; this statement is a "Berwald" version of Hilbert's 4th problem. Further, we investigate geodesic equivalence of Berwald metrics. Theorem 2 gives a system of PDE that has a (nontrivial) solution if and only if the given essentially Berwald metric admits a Riemannian metric that is (nontrivially) geodesically equivalent to it. The system of PDE is linear and of Cauchy–Frobenius type, i.e., the derivatives of unknown functions are explicit expressions of the unknown functions. As an application (Corollary 2), we obtain that geodesic equivalence of an essentially Berwald metric and a Riemannian metric is always affine equivalence provided both metrics are complete.

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