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Title: D'Alembert's functional equation on topological monoids

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We prove that if f is a continuous complex-valued function on the topological monoid M with neutral element e satisfying the functional equation

$$f(xyz) + f(xzy) = 2f(x)f(yz) + 2f(y)f(zx) + 2f(z)f(xy) - 4f(x)f(y)f(z)$$

and $f(e) = 1$, then there is a continuous homomorphism $h : M \to \mathrm{Mat}_2(\mathbb{C})$, the multiplicative monoid of complex 2×2 matrices such that $f = \frac{1}{2}\mathrm{tr}\circ h$. As a consequence we prove that if f is a continuous function on the topological group G satisfying $f(xy)+f(xy^{-1}) = 2f(x)f(y)$ and $f(e) = 1$ then there is a continuous homomorphism $h : G \to \mathrm{SL}_2(\mathbb{C})$ such that $f = \frac{1}{2}\mathrm{tr} \circ h$.

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