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Title: On the Diophantine equation $z^2 = f(x)^2 \pm f(y)^2$

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Let $f \in \mathbb{Q}[X]$ and let us consider a Diophantine equation $z^2 = f(x)^2 \pm f(y)^2$. In this paper, we show that if $\deg f = 2$ and there exists a rational number t such that on the quartic curve $V^2 = f(U)^2 + f(t)^2$ there are infinitely many rational points, then the set of rational parametric solutions of the equation $z^2 = f(x)^2 + f(y)^2$ is non-empty. Without any assumptions we show that the surface related to the Diophantine equation $z^2 = f(x)^2 - f(y)^2$ is unirational over the field $\mathbb{Q}$ in this case. If $\deg f = 3$ and f has the form $f(x) = x(x^2 + ax + b)$ with $a \neq 0$ then both of the equations $z^2 = f(x)^2 \pm f(y)^2$ have infinitely many rational parametric solutions. A similarly result is proved for the equation $z^2 = f(x)^2 - f(y)^2$ with $f(X) = X^3 + aX^2 + b$ and $a \neq 0$.

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