Publ. Math. Debrecen
76/1-2 (2010), 203–218

Real hypersurfaces in complex two-plane Grassmannians with parallel normal Jacobi operator

By IMSOON JEONG (Kyungpook), HEE JIN KIM (Kyungpook) and YOUNG JIN SUH (Kyungpook)

Abstract. In this paper we give a non-existence theorem for Hopf hypersurfaces in complex two-plane Grassmannians $G_2(\mathbb{C}^{m+2})$ with parallel normal Jacobi operator $\bar{R}_N$.

1. Introduction

In the geometry of real hypersurfaces in complex space forms or in quaternionic space forms there have been many characterizations of homogeneous hypersurfaces of type (A_1), (A_2), (B), (C), (D) and (E) in complex projective space $P_m(\mathbb{C})$, of type (A_0),(A_1), (A_2) and (B) in complex hyperbolic space $H_m(\mathbb{C})$ or of type (A_1),(A_2) and (B) in quaternionic projective space $\mathbb{Q}P^m$, which are completely classified by Cecil and Ryan [6], Kimura [9], Kimura and Maeda [10], Berndt [2], Martinez and Pérez [11] respectively.

On the other hand, Jacobi fields along geodesics of a given Riemannian manifold $(\tilde{M}, \tilde{g})$ satisfy an well-known differential equation. This classical differential equation naturally inspires the so-called Jacobi operator. That is, if $\tilde{R}$ is the curvature operator of $\tilde{M}$, and X is any tangent vector field to $\tilde{M}$, the Jacobi operator with respect to X at $p \in \tilde{M}$, $\tilde{R}_X \in \mathrm{End}(T_p\tilde{M})$, is defined by

$$(\tilde{R}_X Y)(p) = (\tilde{R}(Y, X)X)(p)$$

Mathematics Subject Classification: 53C40, 53C15.
Key words and phrases: real hypersurfaces, complex two-plane Grassmannians, parallel normal Jacobi operator, distribution, Hopf hypersurfaces.
This work was supported by grant Proj. No. R17-2008-001-01001-0 from Korea Science & Engineering Foundation.

for any $Y \in T_p\tilde{M}$, becomes a self adjoint endomorphism of the tangent bundle $T\tilde{M}$ of $\tilde{M}$. Clearly, each tangent vector field X to $\tilde{M}$ provides a Jacobi operator with respect to X.

In a complex space form $M_n(c)$, $c \neq 0$, Ki, Pérez, Santos and Suh [8] have investigated real hypersurfaces M in $M_n(c)$ under the condition that $\nabla_\xi S = 0$ and $\nabla_\xi R_\xi = 0$, where S and R_ξ respectively denote the Ricci tensor and the structure Jacobi operator of M in $M_n(c)$. The almost contact structure vector field ξ are defined by $\xi = -JN$, where N denotes a unit normal to M and J a Kaehler structure on $M_n(c)$. Moreover, Pérez, Santos and Suh [13] gave a complete classification of real hypersurfaces in complex projective space whose structure Jacobi operator R_ξ is Lie ξ-parallel, that is, $\mathcal{L}_\xi R_\xi = 0$.

In a quaternionic projective space $\mathbb{Q}P^m$ Pérez and Suh [12] have classified real hypersurfaces in $\mathbb{Q}P^m$ with $\mathfrak{D}^\perp$-parallel curvature tensor $\nabla_{\xi_i} R = 0$, $i = 1, 2, 3$, where R denotes the curvature tensor of M in $\mathbb{Q}P^m$ and $\mathfrak{D}^\perp$ a distribution defined by $\mathfrak{D}^\perp = \text{Span}\{\xi_1, \xi_2, \xi_3\}$. In such a case they are congruent to a tube of radius $\frac{\pi}{4}$ over a totally geodesic $\mathbb{Q}P^k$ in $\mathbb{Q}P^m$, $2 \le k \le m-2$.

The almost contact structure vector fields $\{\xi_1, \xi_2, \xi_3\}$ are defined by $\xi_i = -J_iN$, $i = 1, 2, 3$, where $\{J_1, J_2, J_3\}$ denote a quaternionic Kähler structure of $\mathbb{Q}P^m$ and N a unit normal field of M in $\mathbb{Q}P^m$. In quaternionic space forms Berndt [2] has introduced the notion of normal Jacobi operator

$$\bar{R}_N = \bar{R}(X, N)N \in \text{End } T_xM, \quad x \in M$$

for real hypersurfaces M in a quaternionic projective space $\mathbb{Q}P^m$ or in a quaternionic hyperbolic space $\mathbb{Q}H^m$, where $\bar{R}$ denotes the curvature tensor of $\mathbb{Q}P^m$ and $\mathbb{Q}H^m$ respectively. He [2] has also shown that the curvature adaptedness, that is, the normal Jacobi operator $\bar{R}_N$ commutes with the shape operator A, is equivalent to the fact that the distributions $\mathfrak{D}$ and $\mathfrak{D}^\perp = \text{Span}\{\xi_1, \xi_2, \xi_3\}$ are invariant by the shape operator A of M, where $T_xM = \mathfrak{D} \oplus \mathfrak{D}^\perp$, $x \in M$.

Now let us consider a complex two-plane Grassmannians $G_2(\mathbb{C}^{m+2})$ which consists of all complex 2-dimensional linear subspaces in $\mathbb{C}^{m+2}$. Then the situation for real hypersurfaces in $G_2(\mathbb{C}^{m+1})$ related to the normal Jacobi operator $\bar{R}_N$ is not so simple and will be quite different from the cases mentioned above. In a paper [7] due to Jeong, Suh and Pérez we have classified real hypersurfaces in $G_2(\mathbb{C}^{m+2})$ with commuting normal Jacobi operator, that is, $\bar{R}_N \circ \phi = \phi \circ \bar{R}_N$ or $\bar{R}_N \circ A = A \circ \bar{R}_N$. The normal Jacobi operator $\bar{R}_N$ *commutes* with the shape operator A(or the structure tensor ϕ) of M in $G_2(\mathbb{C}^{m+2})$ means that the eigenspaces of the normal Jacobi operator is *invariant* by the shape operator A(or the structure tensor ϕ).

In this paper we consider a real hypersurface M in complex two-plane Grassmannians $G_2(\mathbb{C}^{m+2})$ with *parallel* normal Jacobi operator, that is, $\nabla_X \bar{R}_N = 0$ for any tangent vector field X on M, where ∇, $\bar{R}$ and N respectively denotes the induced Riemannian connection on M, the curvature tensor of the ambient space $G_2(\mathbb{C}^{m+2})$ and a unit normal vector of M in $G_2(\mathbb{C}^{m+2})$. The normal Jacobi operator $\bar{R}_N$ is *parallel* on M in $G_2(\mathbb{C}^{m+2})$ means that the eigenspaces of the normal Jacobi operator $\bar{R}_N$ is *parallel* along any curve γ in M. Here the eigenspaces of the normal Jacobi operator $\bar{R}_N$ are said to be *parallel* along γ if they are *invariant* with respect to any *parallel displacement* along γ.

The curvature tensor $\bar{R}(X,Y)Z$ for any vector fields X, Y and Z on $G_2(\mathbb{C}^{m+2})$ is explicitly defined in Section 2. Then the normal Jacobi operator $\bar{R}_N$ for the unit normal vector N can be defined from the curvature tensor $\bar{R}(X,N)N$ by putting $Y = Z = N$.

The complex two-plane Grassmannians $G_2(\mathbb{C}^{m+2})$ is known to be the unique compact irreducible Riemannian symmetric space equipped with both a Kähler structure J and a quaternionic Kähler structure $\mathfrak{J}$ not containing J (See Berndt [3]). So, in $G_2(\mathbb{C}^{m+2})$ we have two natural geometric conditions for real hypersurfaces that $[\xi] = \mathrm{Span}\{\xi\}$ or $\mathfrak{D}^{\perp} = \mathrm{Span}\{\xi_1, \xi_2, \xi_3\}$ is invariant under the shape operator. By using such conditions Berndt and Suh [4] have proved the following:

Theorem 1.1. *Let M be a connected real hypersurface in $G_2(\mathbb{C}^{m+2})$, $m \geq 3$. Then both $[\xi]$ and $\mathfrak{D}^{\perp}$ are invariant under the shape operator of M if and only if*

(A) *M is an open part of a tube around a totally geodesic $G_2(\mathbb{C}^{m+1})$ in $G_2(\mathbb{C}^{m+2})$, or*

(B) *m is even, say $m = 2n$, and M is an open part of a tube around a totally geodesic $\mathbb{Q}P^n$ in $G_2(\mathbb{C}^{m+2})$.*

The structure vector field ξ of a real hypersurface M in $G_2(\mathbb{C}^{m+2})$ is said to be a *Reeb* vector field. If the *Reeb* vector field ξ of a real hypersurface M in $G_2(\mathbb{C}^{m+2})$ is invariant by the shape operator, M is said to be a *Hopf hypersurface*. In such a case the integral curves of the *Reeb* vector field ξ are geodesics (See Berndt and Suh [5]). Moreover, the flow generated by the integral curves of the structure vector field ξ for Hopf hypersurfaces in $G_2(\mathbb{C}^{m+2})$ is said to be *geodesic Reeb flow*. Moreover, the corresponding principal curvature α is non-vanishing we say M is with non-vanishing *geodesic Reeb flow*.

Now by putting a unit normal vector N into the curvature tensor $\bar{R}$ of the ambient space $G_2(\mathbb{C}^{m+2})$, we calculate the normal Jacobi operator $\bar{R}_N$ in such a way that

$$\begin{aligned}\bar{R}_N X = \bar{R}(X,N)N &= X + 3\eta(X)\xi + 3\sum_{\nu=1}^{3}\eta_\nu(X)\xi_\nu \\ &\quad - \sum_{\nu=1}^{3}\big\{\eta_\nu(\xi)J_\nu(\phi X + \eta(X)N) - \eta_\nu(\phi X)(\phi_\nu\xi + \eta_\nu(\xi)N)\big\} \\ &= X + 3\eta(X)\xi + 3\sum_{\nu=1}^{3}\eta_\nu(X)\xi_\nu \\ &\quad - \sum_{\nu=1}^{3}\big\{\eta_\nu(\xi)(\phi_\nu\phi X - \eta(X)\xi_\nu) - \eta_\nu(\phi X)\phi_\nu\xi\big\}\end{aligned}$$

for any tangent vector field X on M in $G_2(\mathbb{C}^{m+2})$.

We say that the normal Jacobi operator $\bar{R}_N$ is *parallel* on M if the covariant derivative of the normal Jacobi operator $\bar{R}_N$ identically vanishes, that is, $\nabla_X \bar{R}_N = 0$ for any vector field X on M. Related to such a parallel normal Jacobi operator $\bar{R}_N$ of M in $G_2(\mathbb{C}^{m+2})$, in Section 4 we prove an important theorem for hypersurfaces in $G_2(\mathbb{C}^{m+2})$ as follows:

Theorem 1.2. *Let M be a Hopf hypersurface in $G_2(\mathbb{C}^{m+2})$, $m \geq 3$, with parallel normal Jacobi operator. Then ξ belongs to either the distribution $\mathfrak{D}$ or the distribution $\mathfrak{D}^\perp$.*

In Sections 5 and 6 we respectively prove a non-existence theorem for real hypersurfaces in $G_2(\mathbb{C}^{m+2})$, $m \geq 3$, when the Reeb vector ξ belongs to the distribution $\mathfrak{D}$ or the distribution $\mathfrak{D}^\perp$. Then we assert the following

Theorem 1.3. *There do not exist any Hopf hypersurfaces in $G_2(\mathbb{C}^{m+2})$, $m \geq 3$, with parallel normal Jacobi operator.*

2. Riemannian geometry of $G_2(\mathbb{C}^{m+2})$

In this section we summarize basic material about $G_2(\mathbb{C}^{m+2})$, for details we refer to [3], [4] and [5].

By $G_2(\mathbb{C}^{m+2})$ we denote the set of all complex two-dimensional linear subspaces in $\mathbb{C}^{m+2}$. The special unitary group $G = SU(m+2)$ acts transitively on $G_2(\mathbb{C}^{m+2})$ with stabilizer isomorphic to $K = S(U(2) \times U(m)) \subset G$. Then $G_2(\mathbb{C}^{m+2})$ can be identified with the homogeneous space G/K, which we equip with the unique analytic structure for which the natural action of G on $G_2(\mathbb{C}^{m+2})$

becomes analytic. Denote by $\mathfrak{g}$ and $\mathfrak{k}$ the Lie algebra of G and K, respectively, and by $\mathfrak{m}$ the orthogonal complement of $\mathfrak{k}$ in $\mathfrak{g}$ with respect to the Cartan–Killing form B of $\mathfrak{g}$. Then $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{m}$ is an $Ad(K)$-invariant reductive decomposition of $\mathfrak{g}$.

We put $o = eK$ and identify $T_o G_2(\mathbb{C}^{m+2})$ with $\mathfrak{m}$ in the usual manner. Since B is negative definite on $\mathfrak{g}$, its negative restricted to $\mathfrak{m} \times \mathfrak{m}$ yields a positive definite inner product on $\mathfrak{m}$. By $Ad(K)$-invariance of B this inner product can be extended to a G-invariant Riemannian metric g on $G_2(\mathbb{C}^{m+2})$.

In this way $G_2(\mathbb{C}^{m+2})$ becomes a Riemannian homogeneous space, even a Riemannian symmetric space. For computational reasons we normalize g such that the maximal sectional curvature of $(G_2(\mathbb{C}^{m+2}), g)$ is eight. Since $G_2(\mathbb{C}^3)$ is isometric to the two-dimensional complex projective space $\mathbb{C}P^2$ with constant holomorphic sectional curvature eight we will assume $m \geq 2$ from now on. Note that the isomorphism $Spin(6) \simeq SU(4)$ yields an isometry between $G_2(\mathbb{C}^4)$ and the real Grassmann manifold $G_2^+(\mathbb{R}^6)$ of oriented two-dimensional linear subspaces of $\mathbb{R}^6$.

The Lie algebra $\mathfrak{k}$ has the direct sum decomposition $\mathfrak{k} = \mathfrak{su}(m) \oplus \mathfrak{su}(2) \oplus \mathfrak{R}$, where $\mathfrak{R}$ is the center of $\mathfrak{k}$. Viewing $\mathfrak{k}$ as the holonomy algebra of $G_2(\mathbb{C}^{m+2})$, the center $\mathfrak{R}$ induces a Kähler structure J and the $\mathfrak{su}(2)$-part a quaternionic Kähler structure $\mathfrak{J}$ on $G_2(\mathbb{C}^{m+2})$.

If J_1 is any almost Hermitian structure in $\mathfrak{J}$, then $JJ_1 = J_1 J$, and JJ_1 is a symmetric endomorphism with $(JJ_1)^2 = I$ and $tr(JJ_1) = 0$. This fact will be used frequently throughout this paper.

A canonical local basis J_1, J_2, J_3 of $\mathfrak{J}$ consists of three local almost Hermitian structures J_ν in $\mathfrak{J}$ such that $J_\nu J_{\nu+1} = J_{\nu+2} = -J_{\nu+1} J_\nu$, where the index is taken modulo three. Since $\mathfrak{J}$ is parallel with respect to the Riemannian connection $\bar{\nabla}$ of $(G_2(\mathbb{C}^{m+2}), g)$, there exist for any canonical local basis J_1, J_2, J_3 of $\mathfrak{J}$ three local one-forms q_1, q_2, q_3 such that

$$\bar{\nabla}_X J_\nu = q_{\nu+2}(X) J_{\nu+1} - q_{\nu+1}(X) J_{\nu+2} \tag{1}$$

for all vector fields X on $G_2(\mathbb{C}^{m+2})$.

The Riemannian curvature tensor $\bar{R}$ of $G_2(\mathbb{C}^{m+2})$ is locally given by

$$\begin{aligned} \bar{R}(X,Y)Z = {} & g(Y,Z)X - g(X,Z)Y + g(JY,Z)JX \\ & - g(JX,Z)JY - 2g(JX,Y)JZ \\ & + \sum_{\nu=1}^{3} \{ g(J_\nu Y,Z)J_\nu X - g(J_\nu X,Z)J_\nu Y - 2g(J_\nu X,Y)J_\nu Z \} \\ & + \sum_{\nu=1}^{3} \{ g(J_\nu JY,Z)J_\nu JX - g(J_\nu JX,Z)J_\nu JY \}, \end{aligned} \tag{2}$$

where J_1, J_2, J_3 is any canonical local basis of $\mathfrak{J}$.

3. Some fundamental formulas

In this section we derive some basic formulae from the Codazzi equation for a real hypersurface in $G_2(\mathbb{C}^{m+2})$ (See [4], [5], [14], [15] and [16]).

Let M be a real hypersurface of $G_2(\mathbb{C}^{m+2})$. The induced Riemannian metric on M will also be denoted by g, and ∇ denotes the Riemannian connection of (M, g). Let N be a local unit normal field of M and A the shape operator of M with respect to N. The Kähler structure J of $G_2(\mathbb{C}^{m+2})$ induces on M an almost contact metric structure (ϕ, ξ, η, g). Furthermore, let J_1, J_2, J_3 be a canonical local basis of $\mathfrak{J}$. Then each J_ν induces an almost contact metric structure $(\phi_\nu, \xi_\nu, \eta_\nu, g)$ on M. Using the above expression for $\bar{R}$, the Codazzi equation becomes

$$\begin{aligned}(\nabla_X A)Y - (\nabla_Y A)X = {} & \eta(X)\phi Y - \eta(Y)\phi X - 2g(\phi X, Y)\xi \\ & + \sum_{\nu=1}^{3} \big\{\eta_\nu(X)\phi_\nu Y - \eta_\nu(Y)\phi_\nu X - 2g(\phi_\nu X, Y)\xi_\nu\big\} \\ & + \sum_{\nu=1}^{3} \big\{\eta_\nu(\phi X)\phi_\nu \phi Y - \eta_\nu(\phi Y)\phi_\nu \phi X\big\} \\ & + \sum_{\nu=1}^{3} \big\{\eta(X)\eta_\nu(\phi Y) - \eta(Y)\eta_\nu(\phi X)\big\}\xi_\nu.\end{aligned}$$

The following identities can be proved in a straightforward method and will be used frequently in subsequent calculations:

$$\begin{aligned}\phi_{\nu+1}\xi_\nu = -\xi_{\nu+2}, \quad \phi_\nu \xi_{\nu+1} = \xi_{\nu+2}, \\ \phi\xi_\nu = \phi_\nu \xi, \quad \eta_\nu(\phi X) = \eta(\phi_\nu X), \\ \phi_\nu \phi_{\nu+1} X = \phi_{\nu+2} X + \eta_{\nu+1}(X)\xi_\nu, \\ \phi_{\nu+1}\phi_\nu X = -\phi_{\nu+2} X + \eta_\nu(X)\xi_{\nu+1}.\end{aligned} \tag{3}$$

Now let us put

$$JX = \phi X + \eta(X)N, \quad J_\nu X = \phi_\nu X + \eta_\nu(X)N \tag{4}$$

for any tangent vector X of a real hypersurface M in $G_2(\mathbb{C}^{m+2})$, where N denotes a normal vector of M in $G_2(\mathbb{C}^{m+2})$. Then from this and the formulas (1) and (3) we have that

$$(\nabla_X\phi)Y = \eta(Y)AX - g(AX,Y)\xi, \quad \nabla_X\xi = \phi AX, \tag{5}$$

$$\nabla_X\xi_\nu = q_{\nu+2}(X)\xi_{\nu+1} - q_{\nu+1}(X)\xi_{\nu+2} + \phi_\nu AX, \tag{6}$$

$$(\nabla_X\phi_\nu)Y = -q_{\nu+1}(X)\phi_{\nu+2}Y + q_{\nu+2}(X)\phi_{\nu+1}Y + \eta_\nu(Y)AX - g(AX,Y)\xi_\nu. \tag{7}$$

Moreover, from $JJ_\nu = J_\nu J$, $\nu = 1,2,3$, it follows that

$$\phi\phi_\nu X = \phi_\nu\phi X + \eta_\nu(X)\xi - \eta(X)\xi_\nu. \tag{8}$$

4. Parallel normal Jacobi operator

Now in this section we want to derive the normal Jacobi operator from the curvature tensor $\bar{R}(X,Y)Z$ of complex two-plane Grassmannian $G_2(\mathbb{C}^{m+2})$ given in (2).

Now let us consider a real hypersurface M in $G_2(\mathbb{C}^{m+2})$ with parallel normal Jacobi operator $\bar{R}_N$, that is, $\nabla_X\bar{R}_N = 0$ for any vector field X on M. Then first of all, we write the normal Jacobi operator $\bar{R}_N$, which is given by

$$\begin{aligned}
\bar{R}_N(X) = \bar{R}(X,N)N &= X + 3\eta(X)\xi + 3\sum_{\nu=1}^{3}\eta_\nu(X)\xi_\nu \\
&\quad - \sum_{\nu=1}^{3}\Big\{\eta_\nu(\xi)J_\nu(\phi X + \eta(X)N) - \eta_\nu(\phi X)(\phi_\nu\xi + \eta_\nu(\xi)N)\Big\} \\
&= X + 3\eta(X)\xi + 3\sum_{\nu=1}^{3}\eta_\nu(X)\xi_\nu \\
&\quad - \sum_{\nu=1}^{3}\Big\{\eta_\nu(\xi)(\phi_\nu\phi X - \eta(X)\xi_\nu) - \eta_\nu(\phi X)\phi_\nu\xi\Big\},
\end{aligned}$$

where we have used the following

$$g(J_\nu JN,N) = -g(JN,J_\nu N) = -g(\xi,\xi_\nu) = -\eta_\nu(\xi),$$

$$g(J_\nu JX,N) = g(X,JJ_\nu N) = -g(X,J\xi_\nu) = -g(X,\phi\xi_\nu + \eta(\xi_\nu)N) = -g(X,\phi\xi_\nu)$$

and

$$J_\nu JN = -J_\nu \xi = -\phi_\nu \xi - \eta_\nu(\xi)N.$$

Of course, by (8) we know that the normal Jacobi operator $\bar{R}_N$ could be symmetric endomorphism of T_xM, $x \in M$.

Now let us consider a covariant derivative of the normal Jacobi operator $\bar{R}_N$ along the direction X. Then it is given by

$$\begin{aligned}(\nabla_X \bar{R}_N)Y = {} & 3(\nabla_X\eta)(Y)\xi + 3\eta(Y)\nabla_X\xi + 3\sum_{\nu=1}^{3}(\nabla_X\eta_\nu)(Y)\xi_\nu \\ & + 3\sum_{\nu=1}^{3}\eta_\nu(Y)\nabla_X\xi_\nu - \sum_{\nu=1}^{3}\Big[X(\eta_\nu(\xi))(\phi_\nu\phi Y - \eta(Y)\xi_\nu) \\ & + \eta_\nu(\xi)\big\{(\nabla_X\phi_\nu\phi)Y - (\nabla_X\eta)(Y)\xi_\nu - \eta(Y)\nabla_X\xi_\nu\big\} \\ & - (\nabla_X\eta_\nu)(\phi Y)\phi_\nu\xi - \eta_\nu((\nabla_X\phi)Y)\phi_\nu\xi - \eta_\nu(\phi Y)\nabla_X(\phi_\nu\xi)\Big],\end{aligned}$$

where the formula $X(\eta_\nu(\xi))$ in the right side is given by

$$\begin{aligned}X(\eta_\nu(\xi)) &= g(\nabla_X\xi_\nu, \xi) + g(\xi_\nu, \nabla_X\xi) \\ &= q_{\nu+2}(X)\eta_{\nu+1}(\xi) - q_{\nu+1}(X)\eta_{\nu+2}(\xi) + 2g(\phi_\nu AX, \xi).\end{aligned}$$

From this, together with the formulas given in Section 3, a real hypersurface M in $G_2(\mathbb{C}^{m+2})$ with parallel normal Jacobi operator, that is, $\nabla_X\bar{R}_N = 0$ for any vector field X on M, satisfies the following

$$\begin{aligned}0 = {} & 3g(\phi AX, Y)\xi + 3\eta(Y)\phi AX \\ & + 3\sum_{\nu=1}^{3}\big\{q_{\nu+2}(X)\eta_{\nu+1}(Y) - q_{\nu+1}(X)\eta_{\nu+2}(Y) + g(\phi_\nu AX, Y)\big\}\xi_\nu \\ & + 3\sum_{\nu=1}^{3}\eta_\nu(Y)\big\{q_{\nu+2}(X)\xi_{\nu+1} - q_{\nu+1}(X)\xi_{\nu+2} + \phi_\nu AX\big\} \\ & - \sum_{\nu=1}^{3}\Big[\big\{q_{\nu+2}(X)\eta_{\nu+1}(\xi) - q_{\nu+1}(X)\eta_{\nu+2}(\xi) + 2\eta_\nu(\phi AX)\big\}(\phi_\nu\phi Y - \eta(Y)\xi_\nu) \\ & + \eta_\nu(\xi)\big\{-q_{\nu+1}(X)\phi_{\nu+2}\phi Y + q_{\nu+2}(X)\phi_{\nu+1}\phi Y + \eta_\nu(\phi Y)AX - g(AX, \phi Y)\xi_\nu \\ & + \eta(Y)\phi_\nu AX - g(AX, Y)\phi_\nu\xi - g(\phi AX, Y)\xi_\nu \\ & - \eta(Y)(q_{\nu+2}(X)\xi_{\nu+1} - q_{\nu+1}(X)\xi_{\nu+2} + \phi_\nu AX)\big\} \\ & - \big\{q_{\nu+2}(X)\eta_{\nu+1}(\phi Y) - q_{\nu+1}(X)\eta_{\nu+2}(\phi Y) + g(\phi_\nu AX, \phi Y)\big\}\phi_\nu\xi\end{aligned}$$

$$
\begin{aligned}
&- \bigl\{\eta(Y)\eta_\nu(AX) - g(AX,Y)\eta_\nu(\xi)\bigr\}\phi_\nu\xi \\
&- \eta_\nu(\phi Y)\bigl\{q_{\nu+2}(X)\phi_{\nu+1}\xi - q_{\nu+1}(X)\phi_{\nu+2}\xi \\
&+ \phi_\nu\phi AX - g(AX,\xi)\xi_\nu + \eta(\xi_\nu)AX\bigr\}\Bigr]. \qquad (9)
\end{aligned}
$$

Put $Y = \xi$ in (9), then it follows that

$$
\begin{aligned}
0 = 3\phi AX &+ 3\sum_{\nu=1}^{3}\{q_{\nu+2}(X)\eta_{\nu+1}(\xi) - q_{\nu+1}(X)\eta_{\nu+2}(\xi) + g(\phi_\nu AX,\xi)\}\xi_\nu \\
&+ 3\sum_{\nu=1}^{3}\eta_\nu(\xi)\{q_{\nu+2}(X)\xi_{\nu+1} - q_{\nu+1}(X)\xi_{\nu+2} + \phi_\nu AX\} \\
&+ \sum_{\nu=1}^{3}\{q_{\nu+2}(X)\eta_{\nu+1}(\xi) - q_{\nu+1}(X)\eta_{\nu+2}(\xi) + 2\eta_\nu(\phi AX)\}\xi_\nu \\
&- \sum_{\nu=1}^{3}\eta_\nu(\xi)\phi_\nu AX + \sum_{\nu=1}^{3}\eta_\nu(\xi)\eta(AX)\phi_\nu\xi \\
&+ \sum_{\nu=1}^{3}\eta_\nu(\xi)\{q_{\nu+2}(X)\xi_{\nu+1} - q_{\nu+1}(X)\xi_{\nu+2} + \phi_\nu AX\} \\
&+ \sum_{\nu=1}^{3}\{\eta_\nu(AX) - \eta_\nu(\xi)\eta(AX)\}\phi_\nu\xi.
\end{aligned}
$$

From this we have

$$
\begin{aligned}
0 = 3\phi AX &+ 4\sum_{\nu=1}^{3}\{q_{\nu+2}(X)\eta_{\nu+1}(\xi) - q_{\nu+1}(X)\eta_{\nu+2}(\xi)\}\xi_\nu \\
&+ 4\sum_{\nu=1}^{3}\eta_\nu(\xi)\{q_{\nu+2}(X)\xi_{\nu+1} - q_{\nu+1}(X)\xi_{\nu+2}\} + 5\sum_{\nu=1}^{3}\eta_\nu(\phi AX)\xi_\nu \\
&+ 3\sum_{\nu=1}^{3}\eta_\nu(\xi)\phi_\nu AX + \sum_{\nu=1}^{3}\eta_\nu(AX)\phi_\nu\xi. \qquad (10)
\end{aligned}
$$

On the other hand, we know that

$$
\begin{aligned}
&4\sum_{\nu=1}^{3}\{q_{\nu+2}(X)\eta_{\nu+1}(\xi) - q_{\nu+1}(X)\eta_{\nu+2}(\xi)\}\xi_\nu \\
&\qquad\qquad + 4\sum_{\nu=1}^{3}\eta_\nu(\xi)\{q_{\nu+2}(X)\xi_{\nu+1} - q_{\nu+1}(X)\xi_{\nu+2}\} = 0.
\end{aligned}
$$

Then (10) reduces to

$$0 = 3\phi AX + 5\sum_{\nu=1}^{3}\eta_\nu(\phi AX)\xi_\nu + 3\sum_{\nu=1}^{3}\eta_\nu(\xi)\phi_\nu AX + \sum_{\nu=1}^{3}\eta_\nu(AX)\phi_\nu\xi. \tag{11}$$

If we assume that M is a Hopf, then by putting $X = \xi$ in (11) we have

$$4\alpha\sum_{\nu=1}^{3}\eta_\nu(\xi)\phi_\nu\xi = 0.$$

From this it follows that

$$\alpha = 0 \quad \text{or} \quad \sum_{\nu=1}^{3}\eta_\nu(\xi)\phi_\nu\xi = 0. \tag{12}$$

Now without loss of generality we may put the Reeb vector filed ξ in such a way that

$$\xi = \eta(X_0)X_0 + \eta(\xi_1)\xi_1$$

for some $X_0 \in \mathfrak{D}$ and $\xi_1 \in \mathfrak{D}^\perp$. Then the latter formula of (12) becomes

$$0 = \eta(\xi_1)\phi_1\xi = \eta(X_0)\eta(\xi_1)\phi_1 X_0.$$

This gives that $\eta(X_0) = 0$ or $\eta(\xi_1) = 0$, which means $\xi \in \mathfrak{D}^\perp$ or $\xi \in \mathfrak{D}$. Summing up above facts, we summarize such a situation as follows:

Lemma 4.1. *Let M be a Hopf hypersurface in $G_2(\mathbb{C}^{m+2})$, $m \geq 3$, with parallel normal Jacobi operator. Then the Reeb vector ξ belongs to the distribution $\mathfrak{D}$ or the distribution $\mathfrak{D}^\perp$ unless the geodesic Reeb flow is non-vanishing.*

When the geodesic Reeb flow is vanishing, that is $\alpha = 0$, we can differentiate $A\xi = 0$. Then by a theorem due to Berndt and Suh [5] we know that

$$\sum_{\nu=1}^{3}\eta_\nu(\xi)\phi\xi_\nu = 0.$$

This also gives $\xi \in \mathfrak{D}$ or $\xi \in \mathfrak{D}^\perp$. From this, together with Lemma 4.1, we give a complete proof of Theorem 1.2 mentioned in the introduction.

5. Parallel normal Jacobi operator for $\xi\in\mathfrak{D}$

In this section we want to prove the following proposition

Proposition 5.1. *Let M be a Hopf real hypersurface in $G_2(\mathbb{C}^{m+2})$, $m\geq 3$, with parallel normal Jacobi operator and $\xi\in\mathfrak{D}$. Then $g(A\mathfrak{D},\mathfrak{D}^{\perp})=0$.*

Proof. By Lemma 4.1, let us consider the case that $\xi\in\mathfrak{D}$ in (9). Then we have

$$\begin{aligned}
0 &= 3g(\phi AX,Y)\xi + 3\eta(Y)\phi AX \\
&\quad + 3\sum_{\nu=1}^{3}\{q_{\nu+2}(X)\eta_{\nu+1}(Y) - q_{\nu+1}(X)\eta_{\nu+2}(Y) + g(\phi_\nu AX,Y)\}\xi_\nu \\
&\quad + 3\sum_{\nu=1}^{3}\eta_\nu(Y)\{q_{\nu+2}(X)\xi_{\nu+1} - q_{\nu+1}(X)\xi_{\nu+2} + \phi_\nu AX\} \\
&\quad - \sum_{\nu=1}^{3}\big[2\eta_\nu(\phi AX)(\phi_\nu\phi Y - \eta(Y)\xi_\nu) \\
&\quad - \{q_{\nu+2}(X)\eta_{\nu+1}(\phi Y) - q_{\nu+1}(X)\eta_{\nu+2}(\phi Y) + g(\phi_\nu AX,\phi Y)\}\phi_\nu\xi \\
&\quad - \eta(Y)\eta_\nu(AX)\phi_\nu\xi - \eta_\nu(\phi Y)\{q_{\nu+2}(X)\phi_{\nu+1}\xi - q_{\nu+1}(X)\phi_{\nu+2}\xi \\
&\quad + \phi_\nu\phi AX - g(AX,\xi)\xi_\nu\}\big].
\end{aligned} \tag{13}$$

Then, taking an inner product (13) with ξ, we have

$$\begin{aligned}
0 &= 3g(\phi AX,Y) + 3\sum_{\nu=1}^{3}\eta_\nu(Y)g(\phi_\nu AX,\xi) \\
&\quad - \sum_{\nu=1}^{3}\big[2\eta_\nu(\phi AX)g(\phi_\nu\phi Y,\xi) - \eta_\nu(\phi Y)g(\phi_\nu\phi AX,\xi)\big] \\
&= 3g(\phi AX,Y) + 5\sum_{\nu=1}^{3}\eta_\nu(Y)g(\phi_\nu AX,\xi) + \sum_{\nu=1}^{3}\eta_\nu(\phi Y)g(\phi^2 AX,\xi_\nu) \\
&= 3g(\phi AX,Y) + 5\sum_{\nu=1}^{3}\eta_\nu(Y)g(\phi_\nu AX,\xi) - \sum_{\nu=1}^{3}\eta_\nu(\phi Y)\eta_\nu(AX).
\end{aligned}$$

From this, by putting $Y=\phi Z$ for any $Z\in\mathfrak{D}$, it follows that for any $X\in\mathfrak{D}^{\perp}$ and $\xi\in\mathfrak{D}$

$$3g(AX,Z) = -5\sum_{\nu=1}^{3}\eta_\nu(\phi Z)g(\phi_\nu AX,\xi). \tag{14}$$

Then by putting $Z = \phi\xi_i$ in (14), we have

$$g(AX, \phi\xi_i) = 0$$

for any $i = 1, 2, 3$. From this, together with (14), we assert that $g(AX, Z) = 0$ for any $X \in \mathfrak{D}^\perp$ and $Z \in \mathfrak{D}$. This completes the proof of our Proposition. $\square$

Then by Proposition 5.1 and Theorem 1.1 in the introduction we assert the following

Theorem 5.1. *Let M be a Hopf hypersurface in $G_2(\mathbb{C}^{m+2})$ with parallel normal Jacobi operator and $\xi \in \mathfrak{D}$. Then M is a tube over a totally real and totally geodesic quaternionic projective space $\mathbb{Q}P^n$, $n = 2m$.*

Now let us check whether a real hypersurface of type (B) in $G_2(\mathbb{C}^{m+2})$, that is, a tube over a totally real and totally geodesic $\mathbb{Q}P^n$, satisfy $(\nabla_X \bar{R}_N) = 0$ or not? Corresponding to such a real hypersurface of type (B), we introduce a proposition in Berndt and Suh [4] as follows:

Proposition 5.2. *Let M be a connected real hypersurface of $G_2(\mathbb{C}^{m+2})$. Suppose that $A\mathfrak{D} \subset \mathfrak{D}$, $A\xi = \alpha\xi$, and ξ is tangent to $\mathfrak{D}$. Then the quaternionic dimension m of $G_2(\mathbb{C}^{m+2})$ is even, say $m = 2n$, and M has five distinct constant principal curvatures*

$$\alpha = -2\tan(2r), \quad \beta = 2\cot(2r), \quad \gamma = 0, \quad \lambda = \cot(r), \quad \mu = -\tan(r)$$

with some $r \in (0, \pi/4)$. The corresponding multiplicities are

$$m(\alpha) = 1, \quad m(\beta) = 3 = m(\gamma), \quad m(\lambda) = 4n - 4 = m(\mu)$$

and the corresponding eigenspaces are

$$T_\alpha = \mathbb{R}\xi, \quad T_\beta = \mathfrak{J}J\xi, \quad T_\gamma = \mathfrak{J}\xi, \quad T_\lambda, \quad T_\mu,$$

where

$$T_\lambda \oplus T_\mu = (\mathbb{HC}\xi)^\perp, \quad \mathfrak{J}T_\lambda = T_\lambda, \quad \mathfrak{J}T_\mu = T_\mu, \quad JT_\lambda = T_\mu.$$

Now let us suppose M is of type (B) with parallel normal Jacobi operator $\bar{R}_N$ and $\xi \in \mathfrak{D}$. Then (11) for $\xi \in \mathfrak{D}$ gives

$$0 = 3\phi AX + 5\sum_{\nu=1}^{3} \eta_\nu(\phi AX)\xi_\nu + \sum_{\nu=1}^{3} \eta_\nu(AX)\phi_\nu\xi.$$

From this, by putting $X = \xi_\mu$ and using $A\phi_\nu\xi = 0$ we have

$$0 = 4\beta\phi\xi_\mu.$$

Then it follows that $\beta = 0$. This makes a contradiction. Now, summarizing such a fact, we conclude the following

Theorem 5.2. *There do not exist any Hopf hypersurfaces in* $G_2(\mathbb{C}^{m+2})$, $m \geq 3$, *with parallel normal Jacobi operator and* $\xi \in \mathfrak{D}$.

6. Parallel normal Jacobi operator for $\xi \in \mathfrak{D}^\perp$

In this section, we consider Hopf real hypersurfaces in $G_2(\mathbb{C}^{m+2})$, $m \geq 3$, with parallel normal Jacobi operator and $\xi \in \mathfrak{D}^\perp$. Then (9) gives the following

$$\begin{aligned}
0 &= 3g(\phi AX, Y)\xi + 3\eta(Y)\phi AX \\
&\quad + 3\sum_{\nu=1}^{3}\{q_{\nu+2}(X)\eta_{\nu+1}(Y) - q_{\nu+1}(X)\eta_{\nu+2}(Y) + g(\phi_\nu AX, Y)\}\xi_\nu \\
&\quad + 3\sum_{\nu=1}^{3}\eta_\nu(Y)\{q_{\nu+2}(X)\xi_{\nu+1} - q_{\nu+1}(X)\xi_{\nu+2} + \phi_\nu AX\} \\
&\quad - \big[\{q_2(X) - 2\eta_2(AX)\}(\phi_3\phi Y - \eta(Y)\xi_3) \\
&\quad + \{-q_3(X) + 2\eta_3(AX)\}(\phi_2\phi Y - \eta(Y)\xi_2) \\
&\quad - q_2(X)\phi_3\phi Y + q_3(X)\phi_2\phi Y - g(AX, \phi Y)\xi + \eta(Y)\phi_1 AX \\
&\quad - g(\phi AX, Y)\xi - \eta(Y)(q_3(X)\xi_2 - q_2(X)\xi_3 + \phi_1 AX) \\
&\quad - \{q_1(X)\eta_3(\phi Y) - q_3(X)\eta_1(\phi Y) + g(\phi_2 AX, \phi Y)\}\phi_2\xi \\
&\quad - \{q_2(X)\eta_1(\phi Y) - q_1(X)\eta_2(\phi Y) + g(\phi_3 AX, \phi Y)\}\phi_3\xi \\
&\quad + \eta(Y)\eta_2(AX)\xi_3 - \eta(Y)\eta_3(AX)\xi_2 \\
&\quad - \eta_3(Y)\{q_1(X)\phi_3\xi - q_3(X)\phi_1\xi + \phi_2\phi AX - g(AX, \xi)\xi_2\} \\
&\quad + \eta_2(Y)\{q_2(X)\phi_1\xi - q_1(X)\phi_2\xi + \phi_3\phi AX - g(AX, \xi)\xi_3\}\big].
\end{aligned} \tag{15}$$

Then (15) can be rearranged as follows:

$$\begin{aligned}
0 &= 3g(\phi AX, Y)\xi + 3\eta(Y)\phi AX \\
&\quad + 3\sum_{\nu=1}^{3}\{q_{\nu+2}(X)\eta_{\nu+1}(Y) - q_{\nu+1}(X)\eta_{\nu+2}(Y) + g(\phi_\nu AX, Y)\}\xi_\nu \\
&\quad + 3\sum_{\nu=1}^{3}\eta_\nu(Y)\{q_{\nu+2}(X)\xi_{\nu+1} - q_{\nu+1}(X)\xi_{\nu+2} + \phi_\nu AX\} \\
&\quad - \big[-2\eta_2(AX)\phi_3\phi Y + 2\eta_3(AX)\phi_2\phi Y + g(\phi_2 AX, \phi Y)\xi_3 \\
&\quad - g(\phi_3 AX, \phi Y)\xi_2 + 3\eta(Y)\eta_2(AX)\xi_3 - 3\eta(Y)\eta_3(AX)\xi_2 \\
&\quad - \eta_3(Y)\{\phi_2\phi AX - g(AX, \xi)\xi_2\} + \eta_2(Y)\{\phi_3\phi AX - g(AX, \xi)\xi_3\}\big].
\end{aligned}$$

From this, let us take an inner product with ξ, we have

$$
\begin{aligned}
0 &= 3g(\phi AX, Y) + 3\{q_3(X)\eta_2(Y) - q_2(X)\eta_3(Y) + g(\phi_1 AX, Y)\} \\
&\quad + 3\big\{\eta_3(Y)q_2(X) - \eta_2(Y)q_3(X) + \eta_2(Y)\eta_3(AX) - \eta_3(Y)\eta_2(AX)\big\} \\
&\quad - \{2\eta_2(AX)\eta_3(Y) - 2\eta_3(AX)\eta_2(Y) + \eta_3(Y)\eta_2(AX) - \eta_2(Y)\eta_3(AX)\} \\
&= 3g(\phi AX, Y) + 3g(\phi_1 AX, Y) + 6\eta_2(Y)\eta_3(AX) - 6\eta_3(Y)\eta_2(AX), \qquad (16)
\end{aligned}
$$

where we have used the following formulas

$$
\begin{aligned}
\eta(\phi_3\phi Y) &= -g(\phi_3\xi, \phi Y) = g(\phi\xi_2, Y) = -\eta_3(Y), \\
\eta(\phi_2\phi Y) &= -g(\phi_2\xi, \phi Y) = -g(\phi\xi_3, Y) = -\eta_2(Y), \\
g(\phi_2\phi AX, \xi) &= -g(\phi AX, \phi_2\xi) = g(\phi AX, \xi_3) = -g(AX, \xi_2) = -\eta_2(AX), \\
g(\phi_3\phi AX, \xi) &= -g(\phi AX, \phi_3\xi) = g(AX, \phi\xi_2) = -g(AX, \xi_3) = -\eta_3(AX).
\end{aligned}
$$

Then (16) can be reformed as follows:

$$
0 = g(\phi AX, Y) + g(\phi_1 AX, Y) + 2\eta_2(Y)\eta_3(AX) - 2\eta_3(Y)\eta_2(AX).
$$

From this, by putting $Y = \xi_2$, we have

$$
0 = g(\phi AX, \xi_2) + g(\phi_1 AX, \xi_2) + 2\eta_3(AX) = 2\eta_3(AX) \qquad (17)
$$

for any vector field X on M. Similarly, we are able to assert $\eta_2(AX) = 0$. From this, together with M is Hopf, we assert the following

Proposition 6.1. *Let M be a Hopf hypersurface in $G_2(\mathbb{C}^{m+2})$, $m\geq 3$, with parallel normal Jacobi operator and $\xi\in\mathfrak{D}^\perp$. Then $g(A\mathfrak{D}, \mathfrak{D}^\perp) = 0$.*

From this proposition and together with Theorem 1.1 in the introduction we know that any real hypersurfaces in $G_2(\mathbb{C}^{m+2})$ with parallel normal Jacobi operator $\bar{R}_N$ are congruent to a tube over a totally geodesic $G_2(\mathbb{C}^{m+1})$ in $G_2(\mathbb{C}^{m+2})$.

Now let us check whether real hypersurfaces of type (A) satisfy $\nabla_X \bar{R}_N = 0$ or not? Then we recall a proposition given by Berndt and Suh [4] as follows:

Proposition 6.2. *Let M be a connected real hypersurface of $G_2(\mathbb{C}^{m+2})$. Suppose that $A\mathfrak{D} \subset \mathfrak{D}$, $A\xi = \alpha\xi$, and ξ is tangent to $\mathfrak{D}^\perp$. Let $J_1\in\mathfrak{J}$ be the almost Hermitian structure such that $JN = J_1 N$. Then M has three (if $r = \pi/2\sqrt{8}$) or four (otherwise) distinct constant principal curvatures*

$$
\alpha = \sqrt{8}\cot(\sqrt{8}r), \quad \beta = \sqrt{2}\cot(\sqrt{2}r), \quad \lambda = -\sqrt{2}\tan(\sqrt{2}r), \quad \mu = 0
$$

with some $r \in (0, \pi/\sqrt{8})$. *The corresponding multiplicities are*

$$m(\alpha) = 1, \quad m(\beta) = 2, \quad m(\lambda) = 2m - 2 = m(\mu),$$

and the corresponding eigenspaces we have

$$\begin{aligned} T_\alpha &= \mathbb{R}\xi = \mathbb{R}JN = \mathbb{R}\xi_1, \\ T_\beta &= \mathbb{C}^\perp\xi = \mathbb{C}^\perp N = \mathbb{R}\xi_2 \oplus \mathbb{R}\xi_3, \\ T_\lambda &= \{X \mid X \perp \mathbb{H}\xi, JX = J_1X\}, \\ T_\mu &= \{X \mid X \perp \mathbb{H}\xi, JX = -J_1X\}, \end{aligned}$$

where $\mathbb{R}\xi$, $\mathbb{C}\xi$ *and* $\mathbb{H}\xi$ *respectively denotes real, complex and quaternionic span of the structure vector* ξ *and* $\mathbb{C}^\perp\xi$ *denotes the orthogonal complement of* $\mathbb{C}\xi$ *in* $\mathbb{H}\xi$.

Then, by putting $X = \xi_3$ in (17) and using Proposition 6.2 we have

$$2\eta_3(A\xi_3) = 2\sqrt{2}\cot(\sqrt{2}r) = 0.$$

But $r \in (0, \frac{\pi}{\sqrt{8}})$, on which we know $\cot\sqrt{2}r \neq 0$. This makes a contradiction. Consequently, the normal Jacobi operator $\bar{R}_N$ of such a tube over a totally geodesic $G_2(\mathbb{C}^{m+2})$ can not be parallel. Summing up above facts we conclude the following

Theorem 6.1. *There do not exist any Hopf hypersurfaces in* $G_2(\mathbb{C}^{m+2})$, $m \geq 3$, *with parallel normal Jacobi operator and* $\xi \in \mathfrak{D}^\perp$.

Then by Theorem 1.2, together with Theorems 5.2 and 6.1 in Sections 5 and 6 respectively, we complete the proof of our Theorem 1.3 in the introduction.

References

[1] D. V. Alekseevskii, Compact quaternion spaces, *Func. Anal. Appl.* **2** (1966), 106–114.

[2] J. Berndt, Real hypersurfaces with constant principal curvatures in complex hyperbolic space, *J. Reine Angew. Math.* **395** (1989), 132–141.

[3] J. Berndt, Riemannian geometry of complex two-plane Grassmannians, *Rend. Sem. Mat. Univ. Politec. Torino* **55** (1997), 19–83.

[4] J. Berndt and Y. J. Suh, Real hypersurfaces in complex two-plane Grassmannians, *Monatshefte für Math.* **127** (1999), 1–14.

[5] J. Berndt and Y. J. Suh, Isometric flows on real hypersurfaces in complex two-plane Grassmannians, *Monatshefte für Math.* **137** (2002), 87–98.

[6] T. E. Cecil and P. J. Ryan, Real hypersurfaces in complex space forms, Tight and Taut submanifolds, *MSRI Publ.* **32** (1997), 233–339.

[7] I. Jeong, J. D. Pérez and Y. J. Suh, Real hypersurfaces in complex two-plane Grassmannians with commuting normal Jacobi operator, *Acta Math. Hungarica* **117** (2007), 201–217.

[8] U-H. Ki, J. D. Pérez, F. G. Santos and Y. J. Suh, Real hypersurfaces in complex space forms with ξ-parallel Ricci tensor and structure Jacobi operator, *J. of Korean Math. Soc.* **44** (2007), 307–326.

[9] M. Kimura, Real hypersurfaces and complex submanifolds in complex projective space, *Trans. Amer. Math. Soc.* **296** (1986), 137–149.

[10] M. Kimura and S. Maeda, Lie derivatives on real hypersurfaces in a complex projective space, *Czechoslovak Math. J.* **45(120)** (1995), 135–148.

[11] A. Martinez and J. D. Pérez, Real hypersurfaces in quaternionic projective space, *Ann. Math. Pura Appl.* **145** (1986), 355–384.

[12] J. D. Pérez and Y. J. Suh, Real hypersurfaces of quaternionic projective space satisfying $\nabla_{U_i} R = 0$, *Diff. Geom. and Its Appl.* **7** (1997), 211–217.

[13] J. D. Pérez, F. G. Santos and Y. J. Suh, Real hypersurfaces in complex projective space whose structure Jacobi operator is Lie ξ-parallel, *Diff. Geom. and Its Appl.* **22** (2005), 181–188.

[14] Y. J. Suh, Real hypersurfaces in complex two-plane Grassmannians with parallel shape operator, *Bull. of Austral. Math. Soc.* **67** (2003), 493–502.

[15] Y. J. Suh, Real hypersurfaces in complex two-plane Grassmannians with parallel shape operator II, *J. of Korean Math. Soc.* **41** (2004), 535–565.

[16] Y. J. Suh, Real hypersurfaces in complex two-plane Grassmannians with vanishing Lie derivatives, *Canadian Math. Bull.* **49** (2006), 134–143.

[17] Y. J. Suh, Real hypersurfaces of type B in complex two-plane Grassmannians, *Monatshefte für Math.* **147** (2006), 337–355.

IMSOON JEONG
DEPARTMENT OF MATHEMATICS
KYUNGPOOK NATIONAL UNIVERSITY
TAEGU 702-701
KOREA

E-mail: imsoon.jeong@gmail.com

HEE JIN KIM
DEPARTMENT OF MATHEMATICS
KYUNGPOOK NATIONAL UNIVERSITY
TAEGU 702-701
KOREA

E-mail: hera1oo4@nate.com

YOUNG JIN SUH
DEPARTMENT OF MATHEMATICS
KYUNGPOOK NATIONAL UNIVERSITY
TAEGU 702-701
KOREA

E-mail: yjsuh@knu.ac.kr

(Received January 30, 2009)