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Title: Quasi-central elements and p-nilpotence of finite groups

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Let G be a finite group and let P be a Sylow p-subgroup of G. An element x of G is called quasi-central in G if $\langle x\rangle\langle y\rangle = \langle y\rangle\langle x\rangle$ for each $y \in G$. In this paper, it is proved that G is p-nilpotent if and only if $N_G(P)$ is p-nilpotent and, for all $x \in G \backslash N_G(P)$, one of the following conditions holds: (a) every element of $P \cap P^x \cap G^{\mathcal{N}_p}$ of order p or 4 is quasi-central in P; (b) every element of $P \cap P^x \cap G^{\mathcal{N}_p}$ of order p is quasi-central in P and, if $p = 2$, $P \cap P^x \cap G^{\mathcal{N}_p}$ is quaternion-free; (c) every element of $P \cap P^x \cap G^{\mathcal{N}_p}$ of order p is quasi-central in P and, if $p = 2$, $[\Omega_2(P \cap P^x \cap G^{\mathcal{N}_p}), P] \leq Z(P \cap G^{\mathcal{N}_p})$; (d) every element of $P \cap G^{\mathcal{N}_p}$ of order p is quasi-central in P and, if $p = 2$, $[\Omega_2(P \cap P^x \cap G^{\mathcal{N}_p}), P] \leq \Omega_1(P \cap G^{\mathcal{N}_p})$; (e) $|\Omega_1(P \cap P^x \cap G^{\mathcal{N}_p})| \leq p^{p-1}$ and, if $p = 2$, $P \cap P^x \cap G^{\mathcal{N}_p}$ is quaternion-free; (f) $|\Omega(P \cap P^x \cap G^{\mathcal{N}_p})| \leq p^{p-1}$. That will extend and improve some known related results.

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