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The influence of SNS-permutability of some subgroups on the structure of finite groups

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Abstract. The following concept is introduced: a subgroup H of the group G is said to be SNS-permutable (Subnormal-Sylow-permutable) in G if there is a subnormal subgroup B of G such that $HB = G$ and H permutes with every Sylow subgroup of B. Groups with certain SNS-permutable subgroups of prime power order are studied.

1. Introduction

All groups considered in this paper will be finite; the notation and terminology used in this paper are standard, as in [8]–[10 or [16]. Given a group G, two subgroups H and K of G are said to permute if $HK = KH$, that is, HK is a subgroup of G. A subgroup H of G is said to be S-permutable in G if H permutes with every Sylow subgroup of G. This concept was introduced by Kegel and Deskins in 1962 and has been investigated by many authors, for example, see [1]–[7], [11]–[15], [17]–[25]. In 1998, Ballester-Bolinches and Pedraza-Aguilera extended this concept to S-quasinormally embedded subgroups. A subgroup H of G is S-quasinormally embedded in G if for every Sylow subgroup P of H, there is a S-quasinormal subgroup K in G such that P is also a Sylow subgroup of K. Recently, in [21], Skiba introduced the concept of weakly

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S-permutable subgroup. In [12]–[13], Li, Shen, and other other authors gave the following definition:

Definition 1.1. Let G be a group. A subgroup H of G is said to be an SS-quasinormal subgroup (supplement-Sylow-quasinormal subgroup) of G if there is a supplement B of H in G such that H permutes with every Sylow subgroup of B.

In this paper, we consider another generalization of S-permutable subgroup and give the following definition:

Definition 1.2. Let G be a group. A subgroup H of G is said to be an SNS-permutable subgroup (Subnormal-Sylow-permutable subgroup) of G if there is a subnormal subgroup B such that $HB = G$ and H permutes with every Sylow subgroup of B.

Obviously, every S-permutable subgroup of G is SNS-permutable and every SNS-permutable subgroup is SS-quasinormal. In general, an SNS-permutable subgroup need not be S-permutable. For instance, S_3 is an SNS-permutable subgroup of the symmetric group S_4, but S_3 is not S-permutable. Moreover, an SS-quasinormal subgroup need not be SNS-permutable. For instance, S_4 is an SS-quasinormal subgroup of $PSL(2,7)$, but S_4 is not SNS-permutable in $PSL(2,7)$.

Recall that a formation is a class $\mathcal{F}$ of groups satisfying the following conditions: (i) if $G \in \mathcal{F}$ and $N \trianglelefteq G$, then $G/N \in \mathcal{F}$, and (ii) if N_1, $N_2 \trianglelefteq G$ are such that G/N_1, $G/N_2 \in \mathcal{F}$, then $G/(N_1 \cap N_2) \in \mathcal{F}$. A formation $\mathcal{F}$ is said to be saturated if $G/\Phi(G) \in \mathcal{F}$ implies that $G \in \mathcal{F}$.

We study the influence of the SNS-permutable subgroups on the structure of group G. The main results are as follows:

Theorem 1.1. *Let p be the smallest prime dividing the order of a group G and P a Sylow p-subgroup of G. If P has a subgroup D such that $1 < |D| < |P|$ and all subgroups H of P with order $|H| = |D|$ and with order $2|D|$ (if P is a non-abelian 2-group and $|P : D| > 2$) not having a supersolvable supplement in G are SNS-permutable in G, then G is p-nilpotent.*

Theorem 1.2. *Let $\mathcal{F}$ be a saturated formation containing all supersolvable groups and G a group with a normal subgroup E such that $G/E \in \mathcal{F}$. Suppose that every non-cyclic Sylow subgroup P of $F^*(E)$ has a subgroup D such that $1 < |D| < |P|$ and all subgroups H of P with order $|H| = |D|$ and with order $2|D|$ (if P is a non-abelian 2-group and $|P : D| > 2$) are SNS-permutable in G. Then $G \in \mathcal{F}$.*

2. Preliminaries

Our first result is very useful in proofs using induction arguments. Its proof is a routine checking.

Lemma 2.1. *Suppose that H is SNS-permutable in a group G, $K \leq G$ and N a normal subgroup of G. We have:*

(i) *If $H \leq K$, then H is SNS-permutable in K;*

(ii) *HN/N is SNS-permutable in G/N;*

(iiii) *If $N \leq K$ and K/N is SNS-permutable in G/N, then K is SNS-permutable in G.*

Lemma 2.2. *Suppose that H is a p-subgroup for some prime p and H is not S-permutable in G. Assume that H is SNS-permutable in G. Then G has a normal subgroup M such that $|G : M| = p$ and $G = HM$.*

Proof. By hypothesis G has a subnormal subgroup T such that $HT = G$ and $T \cap H < H$. Hence G has a proper normal subgroup K such that $T \leq K$. Since G/K is a p-group, G has a normal maximal subgroup M such that $HM = G$ and $|G : M| = p$. □

Lemma 2.3. *Let H be a p-subgroup of G. Then the following statements are equivalent:*

(i) *H is S-permutable in G;*

(ii) *$H \leq O_p(G)$ and H is SNS-permutable in G;*

(iii) *$H \leq O_p(G)$ and H is SS-quasinormal in G.*

Proof. We only need to prove that (iii) implies (i). As $H \leq O_p(G)$, it is clear that H permutes with all Sylow p-subgroup of G. By the hypothesis, there is a subgroup $B \leq G$ such that $G = HB$ and $HX = XH$ for all $X \in \mathrm{Syl}(B)$. In particular, if $X = Q \in \mathrm{Syl}_q(B)$, $q \neq p$, then $HQ = QH$. Notice that Q is a Sylow q-subgroup of G. Assume T is another Sylow q-subgroup of G. Then $T = Q^g$ with $g \in G$. Moreover, $g = bh$ with $b \in B$; $h \in H$. Thus $T = Q^g = (Q^b)^h$. As Q^b is another Sylow q-subgroup of B, by the hypothesis, HQ^b is a subgroup of G and from here $H^h(Q^b)^h = HT$ is a subgroup of G. Consequently H permutes with all Sylow q-subgroups of G. Because this holds for all primes $q \neq p$, we have H is S-permutable in G. □

Lemma 2.4. *Let N be an elementary abelian normal subgroup of a group G. Assume that N has a subgroup D such that $1 < |D| < |N|$ and every subgroup H*

of N satisfying $|H| = |D|$ is SNS-permutable in G. Then some maximal subgroup of N is normal in G.

Proof. It follows from Lemma 2.11 of [21] and Lemma 2.3. □

Lemma 2.5. *Let $\mathcal{F}$ be a saturated formation containing all nilpotent groups and let G be a group with solvable $\mathcal{F}$-residual $P = G^{\mathcal{F}}$. Suppose that every maximal subgroup of G not containing P belongs to $\mathcal{F}$. Then P is a p-group for some prime p. In addition, if every cyclic subgroup of P with prime order or order 4 (if $p = 2$ and P is non-abelian) not having a supersolvable supplement in G is SNS-permutable in G, then $|P/\Phi(P)| = p$.*

Proof. By Lemma 2.12 of [21] and Lemma 2.3. □

Lemma 2.6 ([10])**.** *Let G be a group and M a subgroup of G.*

(i) *If M is normal in G, then $F^*(M) \leq F^*(G)$.*

(ii) *$F^*(G) \neq 1$ if $G \neq 1$; in fact, $F^*(G)/F(G) = soc(F(G)C_G(F(G))/F(G))$.*

(iii) *$F^*(F^*(G)) = F^*(G) \geq F(G)$; if $F^*(G)$ is solvable, then $F^*(G) = F(G)$.*

(iv) *Suppose K is a subgroup of G contained in $Z(G)$, then $F^*(G/K) = F^*(G)/K$.*

3. Proofs of the main Theorems

Proof of Theorem 1.1. Assume that the theorem is not true and let G be a counterexample of minimal order. We prove the theorem by the following steps.

(1) $O_{p'}(G) = 1$.

In fact, if $O_{p'}(G) \neq 1$, then we consider the quotient group $G/O_{p'}(G)$. By Lemma 2.1, $G/O_{p'}(G)$ satisfies the hypotheses of the theorem. Thus it follows that $G/O_{p'}(G)$ is p-nilpotent by the choice of G. Hence G is p-nilpotent, a contradiction.

(2) $|D| > p$.

If $|D| = p$, then by Lemma 2.1, G is a minimal non-p-nilpotent group, so $G = [P]Q$, where P, Q are the Sylow p-subgroup and a Sylow q-subgroup of G, respectively. Set $\Phi = \Phi(P)$ and let X/Φ be a subgroup of P/Φ of order p, $x \in X \setminus \Phi$ and $L = \langle x \rangle$. Then L is order p or 4. By the hypotheses, L has a supersolvable supplement in G or is SNS-permutable in G. If L has a supersolvable supplement T in G, then $T \neq G$. So $|G/\Phi : T\Phi/\Phi| = p$. Hence $T\Phi/\Phi \trianglelefteq G/\Phi$

and $P/\Phi \cap T\Phi/\Phi = 1$, it follows that $|P/\Phi| = p$. Therefore P is cyclic and G is p-nilpotent, a contradiction. So L is SNS-permutable in G. By Lemma 2.3, L is S-permutable in G. Moreover, Lemma 2.5 implies that $|P/\Phi| = p$. Consequently, it follows that G is p-nilpotent.

(3) $|P : D| > p$.

If $|P : D| = p$, then by [14. Theorem 1.1], we have that G is p-nilpotent, a contradiction.

(4) All subgroups of P of order $|D|$ and $2|D|$ (if P is a non-abelian 2-group and $|P : D| > 2$) have supersolvable supplement subgroups in G or are S-permutable in G.

Let $H \leq P$ with $|H| = |D|$ or $2|D|$. Assume H has not a supersolvable supplement, therefore it is SNS-permutable in G and it is not S-permutable in G, by Lemma 2.2, there is a normal subgroup M of G such that $|G : M| = p$ and $G = HM$. By (3) and the minimality of G, M is p-nilpotent, and it follows that G is p-nilpotent, a contradiction.

(5) If $N \leq P$ and N is a minimal normal subgroup of G, then $|N| \leq |D|$.

Suppose $|N| > |D|$. Since $N \leq O_p(G)$, N is an elementary abelian group. If a subgroup H of N of order $|D|$ has a supersolvable supplement T in G, then $G = HT = NT$. Hence $N \cap T \trianglelefteq G$. By minimality of N, we have that $N \cap T = 1$ or $N \cap T = N$. If $N \cap T = 1$, then $N = N \cap HT = H(N \cap T) = H$, a contradiction. Thus $N \cap T = N$ and $G = NT = T$, this is also a contradiction. Hence all subgroups of N of order $|D|$ are SNS-permutable. By Lemma 2.2, some maximal subgroup N_1 of N is normal in G. It follows from the minimality of N that $N_1 = 1$, thus $|N| = |D| = p$, a contradiction.

(6) If $N \leq P$ and N is a minimal normal subgroup of G, then G/N is p-nilpotent.

Suppose $|N| < |D|$. By Lemma 2.1 and the minimality of G, G/N is p-nilpotent. By (5), we have $|N| = |D|$. Let $N \leq K \leq P$ with $|K/N| = p$. By (2), N is non-cyclic, so K is also non-cyclic, it follows that K has a maximal subgroup $L \neq N$ and $K = LN$. If L has a supersolvable supplement in G, then K has a supersolvable supplement in G and G/N would be p-nilpotent. So we may assume that L is S-permutable in G, and then $K/N = LN/N$ is S-permutable in G/N. If P/N is abelian, then G/N satisfies the hypothesis. Next suppose that that P/N is a non-abelian 2-group. Hence every subgroup of P of order $2|D|$ not having a supersolvable supplement in G is S-permutable in G. In this case one can show as above that every subgroup X of P containing N and such that $|X : N| = 4$

either has a supersolvable supplement in G or is S-permutable in G. Therefore G/N also satisfies the hypothesis.

(7) $O_p(G) = 1$.

If $O_p(G) \neq 1$, then we can find a minimal normal subgroup N of G contained in $O_p(G)$. By (6), there exists a unique minimal normal subgroup of G, N say (notice that p-nilpotent groups are a saturated formation). Moreover N is not contained in $\Phi(G)$. Therefore $N = O_p(G)$ and there is a maximal subgroup M of G such that $G = NM$, $M \cap N = 1$.

Then by (4) every subgroup H of P satisfying $|H| = |D|$ and not having a supersolvable supplement in G is S-permutable. Since every S-permutable subgroup of G is contained in $O_p(G) = N$, it follows that every subgroup H of P different from N satisfying $|H| = |D|$ has a supersolvable supplement in G. Therefore every maximal subgroup of P has a supersolvable supplement in G, which contradicts Lemma 2.2 of [21]. Thus we have (7).

(8) The final contradiction.

Let H be a subgroup of P of order $|D|$. If H is S-permutable, then $H \leq O_p(G) = 1$, a contradiction. Therefore all subgroups of P of order $|D|$ have supersolvable supplement in G and by Lemma 2.2 of [21], G is p-nilpotent, a contradiction. □

Corollary 3.1. *Let G be a group. If, for every prime p dividing the order of G and $P \in Syl_p(G)$, P has a subgroup D such that $1 < |D| < |P|$ and all subgroups H of P with order $|H| = |D|$ and with order $2|D|$ (if P is a non-abelian 2-group and $|P : D| > 2$) not having a supersolvable supplement in G are SNS-permutable in G, then G has the Sylow tower property of supersolvable type.*

Corollary 3.2. *Let p be the smallest prime dividing the order of a group G and P a Sylow p-subgroup of G. If P has a subgroup D such that $1 < |D| < |P|$ and all subgroups H of P with order $|H| = |D|$ and with order $2|D|$ (if P is a non-abelian 2-group and $|P : D| > 2$) not having a supersolvable supplement in G are S-permutable in G, then G is p-nilpotent.*

Theorem 3.3. *Let $\mathcal{F}$ be a saturated formation containing all supersolvable groups and G a group with a normal subgroup E such that $G/E \in \mathcal{F}$. Suppose that every non-cyclic Sylow subgroup P of E has a subgroup D such that $1 < |D| < |P|$ and all subgroups H of P with order $|H| = |D|$ and with order $2|D|$ (if P is a non-abelian 2-group and $|P : D| > 2$) not having a supersolvable supplement in G are SNS-permutable in G. Then $G \in \mathcal{F}$.*

Proof. Suppose that the theorem is not true and let G be a counterexample of the smallest order. We have the following claims:

(1) $G/Q \in \mathcal{F}$, where Q is a Sylow q-subgroup of E and q is the largest prime dividing $|E|$.

By Lemma 2.1 and Corollary 3.1, E has the Sylow Tower property. Let q be the largest prime dividing $|E|$ and Q a Sylow q-subgroup of E. The fact that E possesses an order Sylow Tower property implies that Q is normal in E. Now Q is characteristic in E and $E \trianglelefteq G$, so $Q \trianglelefteq G$. Furthermore, $(G/Q)/(E/Q) \cong G/E \in \mathcal{F}$ and Lemma 2.1 shows that G/Q satisfies the conditions of the theorem, thus by the choice of G, $G/Q \in \mathcal{F}$.

(2) Every subgroup H of Q with order $|H| = |D|$ not having a supersolvable supplement in G is S-permutable in G.

By Lemma 2.3, we have (2).

(3) If $N \leq Q$ and N is minimal normal subgroup of G, then $G/N \in \mathcal{F}$.

If either $|N| < |D|$ or $|Q : D| = q$, it is clear. So let $|N| = |D|$ and $|Q : D| > q$. Let $N \leq K \leq Q$ where $|K/N| = q$. By Lemma 2.5, $|D| > q$, it follows that N is non-cyclic, so K is also non-cyclic. Hence K has a maximal subgroup $L \neq N$ and $K = LN$. If L has a supersolvable supplement in G then K has a supersolvable supplement in G and G/N would be supersolvable, therefore it would be an $\mathcal{F}$-group. So L is S-permutable in G. Therefore $K/N = LN/N$ is S-permutable in G/N. Consequently, G/N satisfies the hypothesis, as desired.

(4) Final contradiction.

Let N be a minimal normal subgroup of G contained in Q. Applying (3) and the fact that $\mathcal{F}$ is a saturated formation, we obtain that N is the only minimal normal subgroup of G contained in Q and $\Phi(Q) = 1$. Moreover, $N \nsubseteq \Phi(G)$. Therefore, G has a maximal subgroup M such that $G = MN$ and $M \cap N = 1$. On the other hand, $\Phi(Q) = 1$ implies that $Q \cap M$ is normalized by N and M, hence the uniqueness of N yields $N = Q$. But by Lemma 2.4 it is impossible, because Q is a minimal normal subgroup of G. This contradiction completes the proof of this theorem. □

By Theorem 1.3 of [21] and Lemma 2.3, we have:

Corollary 3.4. *Let $\mathcal{F}$ be a saturated formation containing all supersoluble groups and G a group with a solvable normal subgroup E such that $G/E \in \mathcal{F}$ Suppose that every non-cyclic Sylow subgroup P of $F(E)$ has a subgroup D such that $1 < |D| < |P|$ and all subgroups H of P with order $|H| = |D|$ and with order $2|D|$ (if P is a non-abelian 2-group and $|P : D| > 2$) are SNS-permutable in G. Then $G \in \mathcal{F}$.*

Theorem 3.5. *Let G be a group with a normal subgroup E such that G/E is supersolvable, Suppose that every non-cyclic Sylow subgroup P of $F^*(E)$ has a subgroup D such that $1 < |D| < |P|$ and all subgroups H of P with order $|H| = |D|$ and with order $2|D|$ (if P is a non-abelian 2-group and $|P : D| > 2$) are SNS-permutable in G. Then G is supersolvable.*

Proof. Suppose that the theorem is false and let G be a counterexample of smallest order, then we have:

(1) Every proper normal subgroup of G containing $F^*(E)$ is supersolvable.

If N is a proper normal subgroup of G containing $F^*(E)$, we have that $N/N \cap E \cong NE/E$ is supersolvable. By Lemma 2.6, $F^*(E) = F^*(F^*(E)) \leq F^*(E \cap N) \leq F^*(E)$, so $F^*(E \cap N) = F^*(E)$. By Lemma 2.1, $(N, N \cap E)$ satisfy the hypotheses of the theorem, thus the minimal choice of G implies that N is supersolvable.

(2) $E = G$, and $F^*(E) = F(G) < G$.

If $E < G$, then E is supersolvable by (1). In particular, E is solvable, so G is solvable and $F^*(E) = F(E)$. It follows that G is supersolvable by applying Corollary 3.4, a contradiction. If $F^*(G) = G$, then G is supersolvable by Theorem 3.3, a contradiction. Thus $F^*(G) < G$ and $F^*(G)$ is supersolvable by (1), it follows that $F^*(E) = F^*(G) = F(G)$ by Lemma 2.6.

(3) Final contradiction.

Let P be a Sylow p-subgroup of $F(G)$, for some prime p, and let P_1 be an arbitrary subgroup of P of order $|D|$. Then $P_1 \trianglelefteq\trianglelefteq P \trianglelefteq F(G) \trianglelefteq G$. By the hypotheses, P_1 is SNS-permutable in G. So P_1 is S-permutable in G by Lemma 2.3. Thus all subgroups of P of order $|D|$ are S-permutable in G. Applying Corollary 3.4, G is supersolvable, the final contradiction. □

Proof of Theorem 1.2. By Lemma 2.1, we have that all subgroups of any Sylow subgroup of order $|D|$ of $F^*(E)$ are SNS-permutable in E, so Theorem 3.5 implies that E is supersolvable. Hence $F^*(E) = F(E)$. Let P be a Sylow p-subgroup of $F(E)$, for some prime p, and let H be an arbitrary subgroup of order $|D|$ of P. Since P is normal in G, it follows that H is subnormal in G. By the hypotheses, H is SNS-permutable in G. So H is S-permutable in G by Lemma 2.3. Thus all subgroups of P of order $|D|$ are S-permutable in G. Applying Corollary 3.4, G belongs to $\mathcal{F}$. □

In connection with Theorem 1.1 and 1.2 the following natural questions arise:

Remark. Whether Theorem 1.1 and Theorem 1.2 remain true if we replace SNS-permutable by SS-quasinormal or S-quasinormally embedded.

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