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**Title:** On minimal non- $p$ -closed groups and related properties

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Let  $p$  be a prime. A group is called  $p$ -closed if it has a normal Sylow  $p$ -subgroup and it is called  $p$ -exponent closed if the elements of order dividing  $p$  form a subgroup. A group is minimal non- $p$ -closed if it is not  $p$ -closed but its proper subgroups and homomorphic images are  $p$ -closed. Similarly, a group is called minimal non- $p$ -exponent closed if it is not  $p$ -exponent closed but all its proper subgroups and homomorphic images are  $p$ -exponent closed. In this paper we characterize finite minimal non- $p$ -closed groups and investigate the relationship between them and minimal non- $p$ -exponent closed groups. In particular, we show that every minimal non- $p$ -closed group is non- $p$ -exponent closed and that minimal non- $p$ -closed groups and simple minimal non- $p$ -exponent closed groups have cyclic Sylow  $p$ -subgroups. Furthermore, given a prime  $p$ , we describe non- $p$ -exponent closed groups of smallest order and we show that they coincide with non- $p$ -closed groups of smallest order.

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