**Title:** A monotonicity property of Euler's gamma function

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Let

$$\Delta(x) = \frac{\log \Gamma(x+1)}{x} \quad (-1 < x \neq 0), \quad \Delta(0) = -\gamma.$$

For all $n = 0, 1, 2, \ldots$ and $x > -1$, we show that

$$(-1)^n \Delta^{(n+1)}(x) = (n+1)! \int_0^1 u^{n+1} \zeta(n+2, xu+1)\, du,$$

where $\zeta$ denotes the Hurwitz zeta function. This representation implies that $\Delta'$ is completely monotonic on $(-1, \infty)$. This extends a result published in 1996 by GRABNER, TICHY, and ZIMMERMANN, who proved that $\Delta$ is increasing and concave on $(-1, \infty)$.

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