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Title: Nilpotency class of symmetric units of group algebras

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Let F be a field of odd prime characteristic p, G a group, U the group of units in the group algebra FG, and U^+ the subgroup of U generated by the elements of U fixed by the anti-automorphism of FG which inverts all elements of G. It is known that U is nilpotent if G is nilpotent and the commutator subgroup G' has p-power order, and then the nilpotency class of U is at most the order of G'; this bound is attained if and only if G' is cyclic and not a Sylow subgroup of G. Adalbert Bovdi and János Kurdics proved the 'if' part of this last statement by exhibiting a nontrivial commutator of the relevant weight. For the special case when G is a nonabelian torsion group (so G' cannot possibly be a Sylow subgroup), the present paper identifies such a commutator in U^+, showing (Theorem 1) that the same bound is attained even by the nilpotency class of this subgroup. We do not know what happens when G' is not a Sylow subgroup but G is not torsion. It can happen that U^+ is nilpotent even though U is not. The torsion groups G which allow this are known (from the work of Gregory T. Lee) to be precisely the direct products of a finite p-group P, a quaternion group Q of order 8, and an elementary abelian 2-group. Theorem 2: in this case, the nilpotency class of U^+ is strictly smaller than the nilpotency index of the augmentation ideal of the group algebra FP, and this bound is attained whenever P is a powerful p-group. The nonabelian group P of order 27 and exponent 3 is not powerful, yet the $G = P \times Q$ formed with this P also leads to a U^+ attaining the general bound, so here a necessary and sufficient condition remains elusive.

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