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**Title:**  $q$ -series: dimension estimates, linear independence

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Entire transcendental solutions  $f$  of functional equations  $f(q^m z) = R_0(z)f(z) + R_1(z)$  with polynomial coefficients  $R_0, R_1$  are arithmetically studied. The purpose of this note is to report on recent progress on lower bounds for the dimension of the  $K$ -vector space generated by 1 and the values of these  $f$  and their derivatives at  $m$  successive powers of  $q$ , where  $K$  is  $\mathbb{Q}$  or an imaginary quadratic number field. In favorable circumstances, linear independence can be obtained, even in a quantitative form.

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