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Title: Geometric group theory and arithmetic diameter

Author(s): Melvyn B. Nathanson

Let X be a group with identity e, let A be an infinite set of generators for X, and let (X, d_A) be the metric space with the word metric d_A induced by A. If the diameter of the space is infinite, then for every positive integer h there are infinitely many elements $x \in X$ with $d_A(e, x) = h$. It is proved that if $\mathcal{P}$ is a nonempty finite set of prime numbers and A is the set of positive integers whose prime factors all belong to $\mathcal{P}$, then the metric space $(\mathbf{Z}, d_A)$ has infinite diameter. Let $\lambda_A(h)$ denote the smallest positive integer x with $d_A(e, x) = h$. It is an open problem to compute $\lambda_A(h)$ and estimate its growth rate.

Address:
Melvyn B. Nathanson
Lehman College (CUNY)
Bronx, NY 10468
and CUNY Graduate Center
New York, NY 10016
USA