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**Title:** On harmonic numbers and Lucas sequences

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Harmonic numbers  $H_k = \sum_{0 < j \leq k} 1/j$  ( $k = 0, 1, 2, \dots$ ) arise naturally in many fields of mathematics. In this paper we initiate the study of congruences involving both harmonic numbers and Lucas sequences. One of our three theorems is as follows: Let  $u_0 = 0$ ,  $u_1 = 1$ , and  $u_{n+1} = u_n - 4u_{n-1}$  for  $n = 1, 2, 3, \dots$ . Then, for any prime  $p > 5$  we have

$$\sum_{k=0}^{p-1} H_k 2^k u_k \equiv 0 \pmod{p},$$

where  $\equiv 0$  if  $p \equiv 1, 2, 4, 8 \pmod{15}$ , and  $\equiv 1$  otherwise.

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