Year: 2012 **Vol.: 80** **Fasc.: 3-4**

Title: Lattice-like translation ball packings in Nil space

Author(s): Jenő Szirmai

Nil geometry is one of the eight homogeneous Thurston 3-geometries: $\mathbf{E}^3, \mathbf{S}^3, \mathbf{H}^3, \mathbf{S}^2 \times \mathbf{R}, \mathbf{H}^2 \times \mathbf{R}$, $\widetilde{\mathbf{SL_2R}}$, $\mathbf{Nil}, \mathbf{Sol}$. **Nil** can be derived from W. Heisenberg's famous real matrix group. The notion of *translation curve* and translation ball can be introduced by initiative of E. Molnár (see [MS], [MSz], [Sz10]). P. Scott in [S] defined **Nil** lattices to which *lattice-like translation ball packings* can be defined. In our work we will use the projective model of **Nil** geometry introduced by E. Molnár in [M97]. In this paper we have studied the translation balls of **Nil** space and computed their volume. Moreover, we have proved in Theorems 4.1–4.2 that the density of the optimal lattice-like translation ball packing for every natural lattice parameter $1 \leq k \in \mathbb{N}$ is in interval $(0.7808, 0.7889)$ and if $r \in (0, r_d]$ ($r_d \approx 0.7456$) then the optimal density is $\delta_\Gamma^{opt} \approx 0.7808$. Meanwhile we can apply a nice general estimate of L. Fejes Tóth [LFT] in our Theorem 4.2. From Corollary 4.2 we shall see that the kissing number of the lattice-like ball packings is less than or equal to 14 and the optimal ball packing is realizable in case of equality. We formulate a conjecture for δ_Γ^{opt}, where the density of the conjectural densest packing is $\delta_\Gamma^{opt} \approx 0.7808$ for lattice parameter $k = 1$, larger than the Euclidean one ($\frac{\pi}{\sqrt{18}} \approx 0.74048$), but less than the density of the densest lattice-like *geodesic* ball packing in **Nil** space known till now [Sz07]. The kissing number of the translation balls in that packing is 14 as well.

Address:
Jenő Szirmai
Budapest University of Technology and
Economics Institute of Mathematics
Department of Geometry
1521 Budapest, P. O. Box 91
Hungary