

Title: A characterization of Beckenbach families admitting discontinuous Jensen affine functions

Author(s): Michal Lewicki

Let $F : \mathbb{R}^3 \rightarrow \mathbb{R}$ be a continuous function such that $\mathcal{F} := \{\mathbb{R} \ni x \mapsto F(x, a, b) \in \mathbb{R} : a, b \in \mathbb{R}\}$ is a Beckenbach family. Additionally, we assume that for each $a, b \in \mathbb{R}$ the functions $\mathbb{R} \ni x \mapsto F(x, a, b) \in \mathbb{R}$ are monotonic. We show that if there exists a function which is discontinuous at some point and Jensen affine with respect to the family \mathcal{F} , then there exists a strictly increasing and continuous function $h : \mathbb{R} \rightarrow \mathbb{R}$ and continuous $G, H : \mathbb{R}^2 \rightarrow \mathbb{R}$ such that

$$F(u, a, b) = h(G(a, b)u + H(a, b)), \quad (*)$$

for all $u, a, b \in \mathbb{R}$. As a consequence we get an independent proof of theorem of J. Matkowski. Finally, we characterize Beckenbach families of the form (*).

Address:

Michal Lewicki
Institute of Mathematics
Silesian University
Bankowa 14
40-007 Katowice
Poland