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**Title:** On the mod  $p^2$  determination of  $\sum_{k=1}^{p-1} H_k/(k \cdot 2^k)$ : another proof of a conjecture by Sun

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For a positive integer  $n$  let  $H_n = \sum_{k=1}^n 1/k$  be the  $n$ th harmonic number. Z. W. Sun conjectured that for any prime  $p \geq 5$ ,

$$\sum_{k=1}^{p-1} \frac{H_k}{k \cdot 2^k} \equiv \frac{7}{24} p B_{p-3} \pmod{p^2}.$$

This conjecture is recently confirmed by Z. W. Sun and L. L. Zhao. In this note we give another proof of the above congruence by establishing congruences for all the sums of the form  $\sum_{k=1}^{p-1} 2^{\pm k} H_k^r/k^s \pmod{p^{4-r-s}}$  with  $(r, s) \in \{(1, 1), (1, 2), (2, 1)\}$ .

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