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Title: Practical pretenders

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Following Srinivasan, an integer $n \geq 1$ is called *practical* if every natural number in $[1, n]$ can be written as a sum of distinct divisors of n. This motivates us to define $f(n)$ as the largest integer with the property that all of $1, 2, 3, \ldots, f(n)$ can be written as a sum of distinct divisors of n. (Thus, n is practical precisely when $f(n) \geq n$.) We think of $f(n)$ as measuring the "practicality" of n; large values of f correspond to numbers n which we term *practical pretenders*. Our first theorem describes the distribution of these impostors: Uniformly for $4 \leq y \leq x$,

$$\#\{n \leq x : f(n) \geq y\} \asymp \frac{x}{\log y}.$$

This generalizes Saias's result that the count of practical numbers in $[1, x]$ is $\asymp \frac{x}{\log x}$.

Next, we investigate the maximal order of f when restricted to non-practical inputs. Strengthening a theorem of Hausman and Shapiro, we show that every $n > 3$ for which

$$f(n) \geq \sqrt{e^{\gamma} n \log \log n}$$

is a practical number.

Finally, we study the range of f. Call a number m belonging to the range of f an *additive endpoint*. We show that for each fixed $A > 0$ and $\epsilon > 0$, the number of additive endpoints in $[1, x]$ is eventually smaller than $x/(\log x)^A$ but larger than $x^{1-\epsilon}$.

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