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Title: Rational points in geometric progressions on certain hyperelliptic curves

Author(s): Andrew Bremner and Maciej Ulas

We pose a simple Diophantine problem which may be expressed in the language of geometry. Let C be a hyperelliptic curve given by the equation $y^2 = f(x)$, where $f \in Z[x]$ is without multiple roots. We say that points $P_i = (x_i, y_i) \in C(Q)$ for $i = 1, 2, \ldots, k$, are in geometric progression if the numbers x_i for $i = 1, 2, \ldots, k$, are in geometric progression.

Let $n \geq 3$ be a given integer. In this paper we show that there exist polynomials $a, b \in \mathbb{Z}[t]$ such that on the curve $y^2 = a(t)x^n + b(t)$ (defined over the field $\mathbb{Q}(t)$) we can find four points in geometric progression. In particular this result generalizes earlier results of Berczes and Ziegler concerning the existence of geometric progressions on Pell type quadrics $y^2 = ax^2 + b$. We also investigate for fixed $b \in \mathbb{Z}$, when there can exist rationals y_i, $i = 1, \ldots, 4$, with $\{y_i^2 - b\}$ forming a geometric progression, with particular attention to the case $b = 1$. Finally, we show that there exist infinitely many parabolas $y^2 = ax + b$ which contain five points in geometric progression.

Address:
Andrew Bremner
School of Mathematical
and Statistical Sciences
Arizona State University
Tempe AZ 85287-1804
USA

Address:
Maciej Ulas
Jagiellonian University
Faculty of Mathematics
and Computer Science
Institute of Mathematics
Łojasiewicza 6
30-348 Kraków
Poland