

Year: 2013

Vol.: 83

Fasc.: 1-2

**Title:** The shuffle variant of Terai's conjecture on exponential Diophantine equations

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Let  $p, q$  and  $r$  be positive integers with  $p, q, r \geq 2$ , and let  $a, b$  and  $c$  be pair-wise relatively prime positive integers such that  $a^p + b^q = c^r$ . Terai's conjecture states that apart from a handful of exceptions, the exponential Diophantine equation  $a^x + b^y = c^z$  in positive integers  $x, y$  and  $z$ , has the unique solution  $(x, y, z) = (p, q, r)$ . In this paper we consider a similar problem (which we call the shuffle variant of Terai's problem). Our problem states that apart from a handful of exceptions, the exponential Diophantine equation  $c^x + b^y = a^z$  in positive integers  $x, y$  and  $z$ , has the unique solution  $(x, y, z) = (1, 1, p)$  if  $q = r = 2$  and  $c = b + 1$ , and no solutions otherwise. We establish several results on our problem by the theory of linear forms in two archimedean and non-archimedean logarithms with various elementary techniques. In particular we prove that the shuffle variant of Terai's problem is true if  $q = r = 2$  and  $c = b + 1$ .

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