

Year: 2013

Vol.: 83

Fasc.: 1-2

Title: On the powers of integers and conductors of quadratic fields

Author(s): Nihal Bircan and Michael E. Pohst

We consider non-zero integers of the maximal order $\mathcal{O} = \mathcal{O}_F$ of the quadratic field $F = \mathbb{Q}(\sqrt{d})$ where $d \in \mathbb{Z}$ is square-free. Let p be an odd prime and $0 \neq \alpha \in \mathcal{O}_F$. Using the embedding into $\mathrm{GL}(2, \mathbb{R})$ we obtain bounds for the first $\nu \in \mathbb{N}$ such that $\alpha^\nu \equiv 1 \pmod{p}$. For a conductor f , we then study the smallest positive integer $n = n(f)$ such that $\alpha^n \in \mathcal{O}_f$. We obtain bounds for $n(f)$ and for $n(fp^k)$. The most interesting case is where α is the fundamental unit of $\mathbb{Q}(\sqrt{d})$.

Address:

Nihal Bircan
Berlin University of Technology
Institute for Mathematics, MA 3-2
Strasse des 17. Juni 136
D-10623 Berlin
Germany

Address:

Michael E. Pohst
Berlin University of Technology
Institute for Mathematics, MA 3-2
Strasse des 17. Juni 136
D-10623 Berlin
Germany