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Title: Characterization of the convergence of weighted averages of double sequences of numbers and functions in two variables

Author(s): Ferenc Móricz and Ulrich Stadtmüller

Let $(p_j \geq 0)$ and $(q_j \geq 0)$ be two sequences of weights such that

$$P_m := \sum_{k=1}^{m} p_k \to \infty \text{ as } m \to \infty \quad \text{and} \quad Q_n := \sum_{k=1}^{n} q_k \to \infty \text{ as } n \to \infty.$$

Weighted averages of a complex-valued double sequence $(c_{j,k} : (j,k) \in N^2)$ are defined by

$$\sigma_{m,n} := \frac{1}{P_m Q_n} \sum_{j=1}^{m} \sum_{k=1}^{n} p_j \, q_k \, c_{j,k}$$

for large enough m, n where $P_m \cdot Q_n > 0$.

Furthermore, let $p, q : \bar{R}_+ \to \bar{R}_+$ be two weight functions such that

$$P(s) := \int_0^s p(u) \, du \to \infty \text{ as } s \to \infty \quad \text{and} \quad Q(t) := \int_0^t q(u) \, du \to \infty \text{ as } t \to \infty.$$

The weighted averages of a measurable function $f : \bar{R}_+^2 \to C$ are defined when the product $f \, p \, q$ is locally integrable on $\bar{R}_+^2$ as follows

$$\sigma(s,t) := \frac{1}{P(s) \, Q(t)} \int_0^s \int_0^t f(u,v) p(u) q(v) \, du dv$$

for large enough s, t where $P(s)Q(t) > 0$.

Under fairly general conditions imposed on the weights, we give necessary and sufficient conditions in order that the finite limits $\sigma_{mn} \to L$ as $m, n \to \infty$ and $\sigma(s,t) \to L$ as $s, t \to \infty$ exist, respectively. These characterizations may find applications in Probability.

Address:
Ferenc Móricz
University of Szeged
Bolyai Institute
Aradi vértanúk tere 1
H-6720 Szeged
Hungary

Address:
Ulrich Stadtmüller
Universität Ulm
Abt. Zahlentheorie und
Wahrscheinlichkeitstheorie
D-89069 Ulm
Germany