

Title: Arithmetic progressions and Pellian equations

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We consider arithmetic progressions on Pellian equations $x^2 - dy^2 = m$, i.e. integral solutions such that the y -coordinates are in arithmetic progression. We construct explicit infinite families of d, m for which there exists a five-term arithmetic progression in the y -coordinate, and we prove the existence of infinitely many pairs d, m parametrized by points of an elliptic curve of positive rank for which the corresponding Pellian equations have solutions whose y -component form a six-term arithmetic progression. Then we exhibit many six-term progressions whose elements are the y -components of solutions for an equation of the form $x^2 - dy^2 = m$ with small coefficients d, m and also several particular seven-term examples. Finally we show a procedure for searching five-term arithmetic progressions for which there exist a couple of pairs (d_1, m_1) and (d_2, m_2) for which the progression is a solution of the associated Pellian equations. These results extend and complement recent results of DUJELLA, PETHŐ and TADIĆ, and PETHŐ and ZIEGLER.

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