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Title: Volumes and geodesic ball packings to the regular prism tilings in $\widetilde{\mathbf{SL_2R}}$ space

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After having investigated the regular prisms and prism tilings in the $\widetilde{\mathbf{SL_2R}}$ space in a previous work of the second author, we consider the problem of geodesic ball packings related to those tilings and their symmetry groups $\mathbf{pq2_1}$. SLR is one of the eight Thurston geometries that can be derived from the 3-dimensional Lie group of all 2×2 real matrices with determinant one.

In this paper we consider geodesic spheres and balls in $\widetilde{\mathbf{SL_2R}}$ (even in $\mathbf{SL_2R}$), if their radii $\rho \in [0, \frac{\pi}{2})$, and determine their volumes. Moreover, we consider the prisms of the above space, compute their volumes and define the notion of the geodesic ball packing and its density. We develop a procedure to determine the densities of the densest geodesic ball packings for the tilings, or in this paper more precisely, for their generating groups $\mathbf{pq2_1}$ (for integer rotational parameters p, q; $3 \leq p$, $\frac{2p}{p-2} < q$). We look for those parameters p and q above, where the packing density large enough as possible. Now our record is 0.567362 for $(p,q) = (8,10)$. These computations seem to be important, since we do not know optimal ball packing, namely in the hyperbolic space $\mathbf{H}^3$. We know only the density upper bound 0.85326, realized by horoball packing of $\mathbf{H}^3$ to its ideal regular simplex tiling. Surprisingly, for the so-called translation ball packings under the same groups $\mathbf{pq2_1}$ we have got larger density 0.841700 for $(p,q) = (5, 10000 \to \infty)$ close to the above upper bound.

We use for the computation and visualization of the $\widetilde{\mathbf{SL_2R}}$ space its projective model introduced by the first author.

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