Title: Bernstein–Doetsch type theorems for set-valued maps of strongly and approximately convex and concave type

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In this paper, we investigate properties of set-valued mappings that establish connection between the values of this map at two arbitrary points of the domain and the value at their midpoint. Such properties are, for instance, Jensen convexity/concavity, K-Jensen convexity/concavity (where K is the set of nonnegative elements of an ordered vector space), and approximate/strong K-Jensen convexity/concavity. Assuming weak but natural regularity assumptions on the set-valued map, our main purpose is to deduce the convexity/concavity consequences of these properties in the appropriate sense. Our two main theorems will generalize most of the known results in this field, in particular the celebrated Bernstein–Doetsch Theorem from 1915, and thus they offer a unified view of these theories.

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