

Title: Characterizations of peripherally multiplicative mappings between real function algebras

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Let X be a compact Hausdorff space; let $\tau : X \rightarrow X$ be a topological involution; and let $\mathcal{A} \subset C(X, \tau)$ be a real function algebra. Given $f \in \mathcal{A}$, the peripheral spectrum of f is the set $\sigma_\pi(f)$ of spectral values of f of maximum modulus. We demonstrate that if $T_1, T_2 : \mathcal{A} \rightarrow \mathcal{B}$ and $S_1, S_2 : \mathcal{A} \rightarrow \mathcal{A}$ are surjective mappings between real function algebras $\mathcal{A} \subset C(X, \tau)$ and $\mathcal{B} \subset C(Y, \varphi)$ that satisfy $\sigma_\pi(T_1(f)T_2(g)) = \sigma_\pi(S_1(f)S_2(g))$ for all $f, g \in \mathcal{A}$, then there exists a homeomorphism $\psi : \text{Ch}(\mathcal{B}) \rightarrow \text{Ch}(\mathcal{A})$ between the Choquet boundaries such that $(\psi \circ \varphi)(y) = (\tau \circ \psi)(y)$ for all $y \in \text{Ch}(\mathcal{B})$, and there exist functions $\kappa_1, \kappa_2 \in \mathcal{B}$, with $\kappa_1^{-1} = \kappa_2$, such that $T_j(f)(y) = \kappa_j(y)S_j(f)(\psi(y))$ for all $f \in \mathcal{A}$, all $y \in \text{Ch}(\mathcal{B})$, and $j = 1, 2$. As a corollary, it is shown that if either $\text{Ch}(\mathcal{A})$ or $\text{Ch}(\mathcal{B})$ is a minimal boundary (with respect to inclusion) for its corresponding algebra, then the same result holds for surjective mappings $T_1, T_2 : \mathcal{A} \rightarrow \mathcal{B}$ and $S_1, S_2 : \mathcal{A} \rightarrow \mathcal{A}$ that satisfy $\sigma_\pi(T_1(f)T_2(g)) \cap \sigma_\pi(S_1(f)S_2(g)) \neq \emptyset$ for all $f, g \in \mathcal{A}$.

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