

**Title:** Characterization of additive maps  $\xi$ -Lie derivable at zero on von Neumann Algebras

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Let  $\mathcal{M}$  be any von Neumann algebra with the center  $\mathcal{Z}(\mathcal{M})$ . For any scalar  $\xi$ , denote by  $[A, B]_{\xi} = AB - \xi BA$  the  $\xi$ -Lie product of  $A, B \in \mathcal{M}$ . Assume that  $L : \mathcal{M} \rightarrow \mathcal{M}$  is an additive map. It is shown that, if  $\mathcal{M}$  has no central summands of type  $I_1$  or type  $I_2$ , then  $L$  satisfies  $L([A, B]) = [L(A), B] + [A, L(B)]$  whenever  $[A, B] = 0$  if and only if there exists an element  $Z_0 \in \mathcal{Z}(\mathcal{M})$ , an additive map  $h : \mathcal{M} \rightarrow \mathcal{Z}(\mathcal{M})$  and an additive derivation  $\varphi : \mathcal{M} \rightarrow \mathcal{M}$  such that  $L(A) = \varphi(A) + h(A) + Z_0 A$  for all  $A \in \mathcal{M}$ ; if  $\mathcal{M}$  has no central summands of type  $I_1$ , then  $L$  satisfies  $L([A, B]_{\xi}) = [L(A), B]_{\xi} + [A, L(B)]_{\xi}$  whenever  $[A, B]_{\xi} = 0$  with  $\xi \neq 1$  if and only if  $L(I) \in \mathcal{Z}(\mathcal{M})$  and there exists an additive derivation  $\varphi : \mathcal{M} \rightarrow \mathcal{M}$  such that  $\varphi(\xi A) = \xi \varphi(A)$  and  $L(A) = \varphi(A) + L(I)A$  for all  $A \in \mathcal{M}$ . A result in [22] is improved for prime algebra case.

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