Publ. Math. Debrecen
87/1-2 (2015), 47–55
DOI: 10.5486/PMD.2015.7020

Group algebras with almost minimal Lie nilpotency index

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Abstract. Let K be a field of characteristic $p > 0$ and let G be an arbitrary non-abelian group. It is well known that if KG is Lie nilpotent, then its upper as well as lower Lie nilpotency index is at least $p+1$. Shalev investigated Lie nilpotent group algebras whose Lie nilpotency indices are next lower, namely $2p$ and $3p-1$ for $p \geq 5$ and obtained certain interesting results. The aim of this paper is to classify group algebras KG which are Lie nilpotent having Lie nilpotency indices $2p$, $3p-1$ and $4p-2$. Our proofs are independent of Shalev and are valid for $p = 2$ and 3 as well.

1. Introduction

Let R be an associative ring with $1 \neq 0$. By defining the Lie product as $[x, y] = xy - yx$; $x, y \in R$, the ring R can be regarded as a Lie ring, called the associated Lie ring of R. This Lie ring is denoted by $L(R)$. We set $[x_1, \dots, x_n] = [[x_1, \dots, x_{n-1}], x_n]$, where $x_1, \dots, x_n \in R$. The n-th Lie power $R^{[n]}$ of R is the associative ideal generated by all the Lie commutators $[x_1, \dots, x_n]$, where $x_1, \dots, x_n \in R$ and $R^{[1]} = R$. By induction, we define the n-th strong Lie power $R^{(n)}$ of R as the associative ideal generated by all the Lie commutators $[x, y]$, where $x \in R^{(n-1)}$, $y \in R$ and $R^{(1)} = R$. The ring R is called Lie nilpotent / strongly Lie nilpotent if there exists m such that $R^{[m]} = 0$ / $R^{(m)} = 0$. The ring R is called Lie hypercentral if for each sequence a_i of elements of R, there exists some n such that $[a_1, \dots, a_n] = 0$. The minimal integers m, n such that $R^{[m]} = 0$ and $R^{(n)} = 0$ are called the Lie nilpotency index and the strong Lie nilpotency index of R and these are denoted by $t_L(R)$ and $t^L(R)$, respectively. If $R = KG$

Mathematics Subject Classification: Primary: 16S34; Secondary: 17B30.
Key words and phrases: group algebras, Lie nilpotency index.

is the group algebra of a group G over a field K of characteristic p, then $L(KG)$ is the associated Lie algebra of the group algebra KG.

It is well known that the following statements are equivalent for a noncommutative group algebra KG (see [4], [12], [15]):

(i) KG is Lie nilpotent;

(ii) KG is strongly Lie nilpotent;

(iii) KG is Lie hypercentral;

(iv) $U(KG)$ is nilpotent;

(v) characteristic of $K = p > 0$, G is nilpotent and its commutator subgroup G' is a finite p-group.

Here $U(R)$ denotes the unit group of R. It is important to note here that Gupta and Levin [11] have given an example of an algebra R which is Lie nilpotent but not strongly Lie nilpotent. It is also proved in the same paper that if C denotes the nilpotency class of the unit group $U(R)$, then $C + 1 \leq t_L(R)$. This gives an upper bound for C. For $R = KG$, where K is a field of characteristic $p > 0$, Du [10] showed that this bound is actually attained when G is a finite p-group. Catino, Siciliano and Spinelli [7] proved that the same holds when KG is a Lie nilpotent group algebra of a torsion group G containing an element of order p. Many important developments on this topic are reported in Bovdi and Kurdics [5].

According to [18, 19] if G is a non-abelian nilpotent group with $|G'| = p^n$, then $p + 1 \leq t_L(KG) \leq t^L(KG) \leq p^n + 1$. Therefore $p + 1$ is the minimal and $p^n + 1$ is the maximal Lie nilpotency index. Lie nilpotent group algebras of maximal and almost maximal Lie nilpotency index, that is, $p^n - p + 2$ have been described in [6] and [2], [3] respectively. A complete description of Lie nilpotent group algebras of minimal Lie nilpotency index is given in [19]. If KG is Lie nilpotent and $p \geq 5$, then $t_L(KG) = t^L(KG) = 2 + (p-1)\sum_{m\geq 1} m\, d_{(m+1)}$ where $p^{d_{(m)}} = |D_{(m),K}(G) : D_{(m+1),K}(G)|$, $m \geq 2$ and $D_{(m),K}(G) = G \cap (1 + KG^{(m)})$, $m \geq 1$, see [1], [16]. Thus $2p$, $3p - 1$ and $4p - 2$ are the next possible minimal values of $t_L(KG)$. We say that a Lie nilpotent group algebra KG has almost minimal Lie nilpotency index if $t_L(KG) = 2p$. In this article we determine group algebras having almost minimal Lie nilpotency index. We also give a complete characterization of group algebras with Lie nilpotency indices $3p - 1$ and $4p - 2$. We remark that Lie nilpotent group algebras with $t_L(KG) = 2p$ and $3p - 1$ have been described by Shalev [17] for $p \geq 5$. We give here a characterization which is valid for $p = 2$ and 3 as well.

2. Preliminaries

Our notations are standard. C_n denotes the cyclic group of order n and $\gamma_i(G)$ is the i-th term of the lower central series of the group G. If G is a finite p-group, then the nilpotency index of the augmentation ideal $\Delta(G)$ of KG is denoted by $t(G)$. The m-th Lie dimension subgroup $D_{(m),K}(G) = G \cap (1 + KG^{(m)})$, $m \geq 1$. If KG is Lie nilpotent, then according to Jennings' theory [16], $t^L(KG) = 2 + (p-1)\sum_{m\geq 1} m\, d_{(m+1)}$. Here $p^{d_{(m)}} = |D_{(m),K}(G) : D_{(m+1),K}(G)|$, $m \geq 2$ and if $|G'| = p^n$, then it is clear that $\sum_{m\geq 2} d_{(m)} = n$. Explicit expressions for the Lie dimension subgroups $D_{(m),K}(G)$ are given in [14, p. 46], namely, $D_{(m),K}(G) = \prod_{(i-1)p^j \geq m-1} \gamma_i(G)^{p^j}$. Since according to [1] for $p \geq 5$, $t_L(KG) = t^L(KG)$, this gives a formula for calculating the Lie nilpotency index. However, for $p = 2, 3$, $t_L(KG)$ and $t^L(KG)$ may not be equal.

We begin with the following results:

Lemma 2.1 ([11]). *Let R be an associative ring with $1 \neq 0$. Then for any $m, n \geq 1$*

(i) $R^{[m]}R^{[n]} \subseteq R^{[m+n-2]}$;

(ii) $\gamma_m(U(R)) \subseteq 1 + R^{[m]}$.

Lemma 2.2 ([13, Lemma 4]). *Let R be an associative ring with $1 \neq 0$. Then for all $x, y, x_i, y_i, z_i \in U(R)$ and $m, k \geq 1$*

(i) $((x, y) - 1)^k \in R^{[k+1]}$;

(ii) $\prod_{i=1}^{k}((x_i, y_i, z_i) - 1)^2 \in R^{[2+3k]}$;

(iii) $\prod_{i=1}^{k}((x_i, y_i) - 1)^2 R^{[m]} \in R^{[m+2k]}$.

Lemma 2.3 ([18]). *Let KG be a Lie nilpotent group algebra of a group G over a field K of characteristic $p > 0$ and let $n = t_L(KG)$. Then for*

(i) *$p \geq n$, G is abelian;*

(ii) *$p < n$, G is a nilpotent group of class at most c where, c is the least positive integer not less than*

  (a) $\sqrt{2(n-1)/(p-1)}$, *if $p > 3$*

  (b) $1 + \sqrt{n-4}$, *if $p = 3$.*

Lemma 2.4 ([5, Theorem 3.2]). *Let K be a field of characteristic $p > 0$ and let G be a nilpotent group such that G' is a finite abelian p-group with invariants $(p^{m_1}, p^{m_2}, \ldots, p^{m_s})$. Then*

(i) $t_L(KG) \geq t(G') + 1 = 2 + \sum_{i=1}^{s}(p^{m_i} - 1)$;

(ii) $t_L(KG) = t^L(KG) = t(G') + 1$, *if* $\gamma_3(G) \subseteq G'^p$.

Lemma 2.5 ([7], [10], [12]). *Let* KG *be a Lie nilpotent group algebra of a torsion group* G *containing an element of order* p *over a field* K *of characteristic* $p > 0$. *Then* $U(KG)$ *is nilpotent of class* $C = t_L(KG) - 1$.

Lemma 2.6. *For any associative ring* R *with* $1 \neq 0$

(i) $R^{[3]}R^{[3]} \subseteq R^{[5]}$;

(ii) *For all* $a_i \in \gamma_3(U(R))$, $\prod_{i=1}^{k}(a_i - 1)^2 \in R^{[2+3k]}$.

Proof. (i) Let $a, b, c, d, e, f \in R$. Then

$$[a, b, c][d, e, f] + [a, b, f][d, e, c]$$
$$= [a, b, [d, e, cf]] - c[a, b, [d, e, f]] - [a, b, [d, e, c]]f + [[a, b, f], [d, e, c]] \in R^{[5]}.$$

Also by [13, Lemma 2(a)]

$$[a, b, c][d, e, f] - [b, c, a][d, e, f] \in R^{[5]}.$$

Thus

$$\begin{aligned}
[c, a, b][d, e, f] &\equiv -[c, a, f][d, e, b] \equiv -[a, f, c][d, e, b] \equiv -[a, f, c][e, b, d] \\
&\equiv [a, f, d][e, b, c] \equiv [f, d, a][e, b, c] \equiv [f, d, a][b, c, e] \\
&\equiv -[f, d, e][b, c, a] \equiv -[d, e, f][b, c, a] \\
&\equiv -[b, c, a][d, e, f] \mod R^{[5]}.
\end{aligned}$$

So $[a, b, c][d, e, f] \in R^{[5]}$.

(ii) Proof is by induction on k and Lemma 2.1. □

Lemma 2.7. *Let* K *be a field of characteristic* 3 *and let* G *be a nilpotent group of class* 3 *such that* G' *is an elementary abelian* 3*-group of order* 3^n *and* $|\gamma_3(G)| = 3^m$, $m \geq 1$. *Then* $t_L(KG) \geq 3 + m + 2n$. *In particular if* $m = 1$, *then* $t_L(KG) = 4 + 2n$.

Proof. Let $a_1, a_2, \dots, a_m$ be independent generators of $\gamma_3(G)$ and let commutators $b_1, b_2, \dots, b_{n-m} \in G'$ be such that

$$\{b_1\gamma_3(G), b_2\gamma_3(G), \dots, b_{n-m}\gamma_3(G)\}$$

form a basis of $G'/\gamma_3(G)$. Then

$$0 \neq \prod_{i=1}^{m}(a_i - 1)^2 \prod_{i=1}^{n-m}(b_i - 1)^2 \in KG^{[2+3m+2(n-m)]}$$

by Lemma 2.2 and Lemma 2.6. So $t_L(KG) \geq 3 + m + 2n$. Also $d_{(2)} = n - m$ and $d_{(3)} = m$. Hence $t^L(KG) = 2+2n+2m$. Thus if $m = 1$, then $t_L(KG) = 4+2n$. □

3. Lie nilpotent group algebras of almost minimal index

Throughout this section G is a non-abelian group and K is a field of positive characteristic p such that KG is Lie nilpotent. We first give classification of Lie nilpotent group algebras of small Lie nilpotency index. Such Lie algebras of index upto 6 have been described in [9].

Theorem 3.1. *Let KG be a Lie nilpotent group algebra of a non-abelian group G over a field K of characteristic $p > 0$ such that $t_L(KG) = 7$ or 9. Then $p = 2$.*

Proof. Let $p = 3$ and $t_L(KG) = 7$. If $|G'| \leq 5$, then $t_L(KG) \leq 6$. So $|G'| > 5$. By Lemma 2.3, G is nilpotent of class at most 3. Thus $\gamma_4(G) = 1$ and G' is abelian. Also the exponent of G' is at most 3. By Lemma 2.4, $G' \cong C_3 \times C_3$. But then by Lemma 2.4 and Lemma 2.7, $t_L(KG) = 6$ or 8.

If $t_L(KG) = 9$, let $x_i \in G$ such that $(x_1, x_2, \ldots, x_i) \in \gamma_i(G) \setminus \gamma_{i+1}(G)$. Then by Lemma 2.1 and Lemma 2.2, $((x_1, x_2) - 1)^2((x_1, x_2, x_3) - 1)^2((x_1, x_2, x_3, x_4) - 1)^2 \in KG^{[11]} = 0$. So again $\gamma_4(G) = 1$ and G' is abelian. Also the exponent of G' is at most 3 and $|G'|$ is at least 8. Now by Lemma 2.4, $G' \cong C_3 \times C_3$ or $G' \cong C_3 \times C_3 \times C_3$. In the first case $t_L(KG) = 6$ or 8. In the second case, let $\gamma_3(G) \neq 1$. Then by Lemma 2.7, $t_L(KG) \geq 10$. So $\gamma_3(G) = 1$, but then $t_L(KG) = 8$ by Lemma 2.4.

For $p \geq 5$, $t_L(KG) = t^L(KG)$ is always even by [1, Theorem 1]. □

Theorem 3.2. *Let KG be a Lie nilpotent group algebra of a non-abelian group G over a field K of characteristic $p \geq 3$ such that $t_L(KG) = 8$. Then $t^L(KG) = 8$.*

Proof. Let $p = 3$. If $t_L(KG) = 8$, then by Lemma 2.3, $\gamma_4(G) = 1$. Also in this case KG is Lie solvable of derived length at most 3. So by [8, Theorem 2.4] $G' \cong C_3 \times C_3 \times C_3$; $\gamma_3(G) = 1$ or $G' \cong C_3 \times C_3$; $\gamma_3(G) \cong C_3$. In both the cases $t^L(KG) = 8$, see [9, Theorem 3.6].

For $p \geq 5$, $t_L(KG) = t^L(KG)$ by [1, Theorem 1]. □

Theorem 3.3. *Let KG be a Lie nilpotent group algebra of a non-abelian group G over a field K of characteristic $p \geq 3$ such that $t_L(KG) = 10$. Then $t^L(KG) = 10$.*

Proof. Let $p = 3$ and $t_L(KG) = 10$. Let $x_i \in G$ such that $(x_1, x_2, \ldots, x_i) \in \gamma_i(G) \setminus \gamma_{i+1}(G)$. Then by Lemma 2.1 and Lemma 2.2, $((x_1, x_2) - 1)^2((x_1, x_2, x_3) - 1)^2 ((x_1, x_2, x_3, x_4) - 1)^2 \in KG^{[11]} = 0$. So $\gamma_4(G) = 1$ and G' is abelian. Also $|G'| \geq 9$ and the exponent of G' is at most 9. By Lemma 2.4, $G' \cong C_9$ or $C_3 \times C_3$ or

$C_3 \times C_3 \times C_3$ or $C_3 \times C_3 \times C_3 \times C_3$. In the second case, $t_L(KG) = 6$ or 8. In the third case, if $|\gamma_3(G)| = 9$, then as has been shown in Lemma 2.7, $t_L(KG) \geq 11$ and if $\gamma_3(G) = 1$, then $t_L(KG) = 8$. Therefore, in this case, $\gamma_3(G) \cong C_3$. In the fourth case, if $\gamma_3(G) \neq 1$, then by Lemma 2.7, $t_L(KG) \geq 12$. Therefore, in this case $\gamma_3(G) = 1$. Thus we have either $G' \cong C_9$ or $G' \cong C_3 \times C_3 \times C_3$; $\gamma_3(G) \cong C_3$; $\gamma_4(G) = 1$ or $G' \cong C_3 \times C_3 \times C_3 \times C_3$; $\gamma_3(G) = 1$. In the first and the third case, $t^L(KG) = 10$ by [6, Theorem 1] and Lemma 2.4. In the second case, $d_{(2)} = 2$, $d_{(3)} = 1$ and $d_{(k)} = 0$ for all $k \geq 4$ so again $t^L(KG) = 10$.

For $p \geq 5$, $t_L(KG) = t^L(KG)$ by [1, Theorem 1]. □

We now give characterizations of Lie nilpotent group algebras of Lie nilpotency indices $2p$, $3p-1$ and $4p-2$.

Theorem 3.4. *Let KG be a Lie nilpotent group algebra of a group G over a field K of characteristic $p > 0$. Then the following are equivalent:*

(i) $t_L(KG) = 2p$;

(ii) $t^L(KG) = 2p$;

(iii) $G' \cong C_p \times C_p$ *and* $\gamma_3(G) = 1$.

Proof. Let $t^L(KG) = 2p$. Then $d_{(2)} + 2d_{(3)} = 2$ and $d_{(k)} = 0$ for all $k \geq 4$. If $d_{(3)} \neq 0$, then $|G'| = p$ and $t^L(KG) = p + 1$. So $d_{(3)} = 0$, $d_{(2)} = 2$ and $|G'| = p^2$. Now if G' is cyclic, then $t^L(KG) = p^2 + 1$. So $G' \cong C_p \times C_p$. Also $D_{(3),K}(G) = D_{(4),K}(G) = \cdots = D_{(2p),K}(G) = 1$. Hence $\gamma_3(G) = 1$. Conversely, if $G' \cong C_p \times C_p$ and central, then $D_{(3),K}(G) = \gamma_3(G)G'^p = 1$. Hence $t^L(KG) = 2 + 2(p-1) = 2p$. Thus statements (ii) and (iii) are equivalent. If $p = 2$ or 3, then $t^L(KG) = t_L(KG)$ follows easily from [9, Theorem 3.7] and for $p \geq 5$ this is a simple consequence of [1, Theorem 1]. □

Theorem 3.5. *Let KG be a Lie nilpotent group algebra of a group G over a field K of characteristic $p > 0$. Then $t^L(KG) = 3p - 1$ if and only if one of the following conditions holds:*

(i) $G' \cong C_p \times C_p \times C_p$ *and* $\gamma_3(G) = 1$;

(ii) $G' \cong C_p \times C_p$, $\gamma_3(G) \cong C_p$ *and* $\gamma_4(G) = 1$;

(iii) $p = 2$, $G' \cong C_4$.

Moreover, $t^L(KG) = 3p - 1$ if and only if $t_L(KG) = 3p - 1$.

Proof. Let $t^L(KG) = 3p - 1$. Then $d_{(2)} + 2d_{(3)} + 3d_{(4)} = 3$ and $d_{(k)} = 0$ for all $k \geq 5$. If $d_{(4)} \neq 0$, then $|G'| = p$ and $t^L(KG) = p + 1$. So $d_{(4)} = 0$ and $D_{(4),K}(G) = D_{(5),K}(G) = \cdots = D_{(3p-1),K}(G) = 1$. Thus G' is a nilpotent group of class at most 3. If $d_{(3)} = 0$, then $d_{(2)} = 3$ and $D_{(3),K}(G) = 1$. So, in this case, the exponent of G' is p, $\gamma_3(G) = 1$ and hence $G' \cong C_p \times C_p \times C_p$. If $d_{(3)} \neq 0$, then $d_{(2)} = d_{(3)} = 1$ and $|G'| = p^2$. Now if G' is cyclic, then $t^L(KG) = p^2 + 1 = 3p - 1$. So $p = 2$, $\gamma_3(G) \subseteq G'^2$ and $\gamma_4(G) = 1$. If G' is not cyclic, then $G' \cong C_p \times C_p$, $\gamma_3(G) \cong C_p$ and $\gamma_4(G) = 1$. Conversely, if condition (i) holds, then $d_{(2)} = 3$ and $d_{(k)} = 0$ for all $k \geq 3$, so $t^L(KG) = 2 + 3(p-1) = 3p - 1$. In case of condition (ii), $d_{(2)} = d_{(3)} = 1$ and $d_{(k)} = 0$ for all $k \geq 4$. So $t^L(KG) = 2 + (p-1)(1+2) = 3p - 1$. For condition (iii), $t^L(KG) = 5 = 3p - 1$ by [6, Theorem 1].

Moreover, if $p = 2$ or 3 and $t^L(KG) = 3p - 1$, then $t_L(KG) = 3p - 1$ follows easily from [9, Theorem 3.3 and Theorem 3.6]. Conversely, if $t_L(KG) = 3p - 1$ and $p = 2$ or 3, then $t^L(KG) = 3p - 1$ by [9, Theorem 3.7] and Theorem 3.2. For $p \geq 5$ this is a simple consequence of [1, Theorem 1]. □

Theorem 3.6. *Let KG be a Lie nilpotent group algebra of a group G over a field K of characteristic $p > 0$. Then $t^L(KG) = 4p - 2$ if and only if one of the following conditions holds:*

(i) $G' \cong C_p \times C_p \times C_p \times C_p$ *and* $\gamma_3(G) = 1$;

(ii) $G' \cong C_p \times C_p \times C_p$, $\gamma_3(G) \cong C_p$ *and* $\gamma_4(G) = 1$;

(iii) $p = 3$ *and* $G' \cong C_9$;

(iv) $p = 2$, $G' \cong C_4 \times C_2$, $\gamma_3(G) = G'^2$ *and* $\gamma_4(G) = 1$.

Moreover, $t^L(KG) = 4p - 2$ *if and only if* $t_L(KG) = 4p - 2$.

Proof. Let $t^L(KG) = 4p - 2$. Then $d_{(2)} + 2d_{(3)} + 3d_{(4)} + 4d_{(5)} = 4$ and $d_{(k)} = 0$ for all $k \geq 6$. If $d_{(5)} \neq 0$, then $|G'| = p$ and $t^L(KG) = p + 1$. So $d_{(5)} = 0$ and $D_{(5),K}(G) = D_{(6),K}(G) = \cdots = D_{(4p-2),K}(G) = 1$. Thus G' is a nilpotent group of class at most 4. If G' is a cyclic group of order p^2, then $t^L(KG) = p^2 + 1 = 4p - 2$ and so $p = 3$. If $d_{(4)} \neq 0$, then $d_{(2)} = 1$ and $|G'| = p^2$. So, in this case, $t^L(KG) = 2p$ or $3p - 1$ by Theorem 3.4 and Theorem 3.5. Thus $d_{(4)} = 0$ and $D_{(4),K}(G) = 1$. For the same reason $d_{(3)} \neq 2$. Now we have two possibilities. Either $d_{(2)} = 4$, $d_{(3)} = 0$ or $d_{(2)} = 2$, $d_{(3)} = 1$. In the first case $G' \cong C_p \times C_p \times C_p \times C_p$ and $\gamma_3(G) = 1$. In the second case $|D_{(3),K}(G)| = p$. Now if $p \neq 2$, then $D_{(4),K}(G) = 1$ leads to $G'^p = 1$. So $G' \cong C_p \times C_p \times C_p$, $\gamma_3(G) \cong C_p$ and $\gamma_4(G) = 1$. But if $p = 2$, then $t^L(KG) = 6$ and we have case (iv) by [9, Theorem 3.4]. Conversely, if case (i) holds, then $d_{(2)} = 4$ and $d_{(k)} = 0$ for all $k \geq 3$. So $t^L(KG) = 2 + 4(p-1) = 4p - 2$. In case (ii), $d_{(2)} = 2$, $d_{(3)} = 1$ and $d_{(k)} = 0$ for all $k \geq 4$ so again $t^L(KG) = 4p - 2$. In cases (iii) and (iv) [6, Theorem 1] and [9, Theorem 3.4] apply, respectively.

Moreover, if $p = 2$ and $t^L(KG) = 4p - 2$, then $t_L(KG) = 4p - 2$ follows easily from [9, Theorem 3.4]. If $p = 3$ and $t^L(KG) = 4p - 2$, then in case (i) we can use Lemma 2.4 and in case (ii) Lemma 2.7 applies. In case (iii) we have [6, Theorem 1]. Conversely, if $t_L(KG) = 4p-2$ and $p = 2$ or 3, then $t^L(KG) = 4p-2$ by [9, Theorem 3.7] and Theorem 3.3. For $p \geq 5$ this is a simple consequence of [1, Theorem 1]. □

As a straightforward application of Lemma 2.5, we obtain:

Corollary 3.7. *Let KG be a Lie nilpotent group algebra of a torsion group G containing an element of order p over a field K of characteristic $p > 0$. Let C be the nilpotency class of $U(KG)$. Then*

(i) *$C = 2p - 1$ if and only if $G' \cong C_p \times C_p$ and $\gamma_3(G) = 1$;*

(ii) *$C = 3p-2$ if and only if G and K satisfy one of the conditions of Theorem 3.5;*

(iii) *$C = 4p-3$ if and only if G and K satisfy one of the conditions of Theorem 3.6.*

Acknowledgements. The author thanks the referees for their suggestions that led to a better presentation of the paper.

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(Received March 24, 2014; revised January 12, 2015)