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Title: Counting invertible sums of squares modulo n and a new generalization of Euler's totient function

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In this paper we introduce and study a family Φ_k of arithmetic functions generalizing Euler's totient function. These functions are given by the number of solutions to the equation $\gcd(x_1^2 + \cdots + x_k^2, n) = 1$ with $x_1, \ldots, x_k \in \mathbb{Z}/n\mathbb{Z}$ which, for $k = 2, 4$ and 8 coincide, respectively, with the number of units in the rings of Gaussian integers, quaternions and octonions over $\mathbb{Z}/n\mathbb{Z}$. We prove that Φ_k is multiplicative for every k, we obtain an explicit formula for $\Phi_k(n)$ in terms of the prime-power decomposition of n and derive an asymptotic formula for $\sum_{n\le x} \Phi_k(n)$. As a tool we investigate the multiplicative arithmetic function that counts the number of solutions to $x_1^2+\cdots+x_k^2 \equiv \lambda \pmod{n}$ for λ coprime to n, thus extending an old result that dealt only with the prime n case.

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