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Title: Automorphic loops arising from module endomorphisms

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A loop is automorphic if all its inner mappings are automorphisms. We construct a large family of automorphic loops as follows. Let R be a commutative ring, V an R-module, $E = \mathrm{End}_R(V)$ the ring of R-endomorphisms of V, and W a subgroup of $(E, +)$ such that $ab = ba$ for every a, $b \in W$ and $1 + a$ is invertible for every $a \in W$. Then $Q_{R,V}(W)$ defined on $W \times V$ by

$$(a, u)(b, v) = (a + b, u(1 + b) + v(1 - a))$$

is an automorphic loop.

A special case occurs when $R = k < K = V$ is a field extension and W is a k-subspace of K such that $k1 \cap W = 0$, naturally embedded into $\mathrm{End}_k(K)$ by $a \mapsto M_a$, $bM_a = ba$. In this case we denote the automorphic loop $Q_{R,V}(W)$ by $Q_{k<K}(W)$.

We call the parameters tame if k is a prime field, W generates K as a field over k, and K is perfect when $\mathrm{char}(k) = 2$. We describe the automorphism groups of tame automorphic loops $Q_{k<K}(W)$, and we solve the isomorphism problem for tame automorphic loops $Q_{k<K}(W)$. A special case solves a problem about automorphic loops of order p^3 posed by Jedlička, Kinyon and Vojtěchovský.

We conclude the paper with a construction of an infinite 2-generated abelian-by-cyclic automorphic loop of prime exponent.

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