Publ. Math. Debrecen
88/3-4 (2016), 517–519
DOI: 10.5486/PMD.2016.7521

Corrigenda: The geometry of a Randers rotational surface

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In our paper [1], amongst other topics, we have determined the structure of the cut locus of a point on a Randers rotational surface by deforming the corresponding Riemannian cut locus using the flow of W. However, without being anything wrong with our proofs, we have started with a wrongly positioned Riemannian first conjugate point of q. After correcting this, our Theorem 1.5 and its proof can be simplified as follows.

Theorem 1.5 (p. 476)**.** *Let $(M, F = \alpha + \beta)$ be a rotational Randers von Mangoldt surface of revolution. Then, for any point $q \neq p$, the Finslerian cut locus $\mathcal{C}_q^{(F)}$ of q is the Jordan arc*

$$\mathcal{C}_q^{(F)} = \{\varphi(s, \tau_q(s)) : s \in [c, \infty)\},$$

where $\varphi(c, \tau_q(c))$ is the first conjugate point of q along the twisted meridian $\varphi(s, \tau_q(s))$.

Proof of Theorem 1.5. (p. 498–500) First of all, observe that from our hypothesis we know that the h-cut locus of q is exactly $\tau_q|_{[c,\infty)}$, where $\tau_q(c)$ is the first h-conjugate point of q along τ_q (see Theorem 7.3.1 in [2]).

We divide our proof in two steps.

At the first step, we will establish the correspondence of h-conjugate points of q along τ_q with the F-conjugate points of q along an F-geodesic from q.

Let $\tilde{x} = \tau_q(c)$ the first h-conjugate point of q along τ_q. Observe that in the case of the Riemannian surface of revolution (M, h), we must have $c > \rho$, because p is the unique pole for h. This is equivalent to saying that $\tilde{x}$ is conjugate to q along τ_q (see [2], [3]).

Recall that $\tilde{x} = \tau_q(c)$ is the first h-conjugate point of q along τ_q means that the Jacobi field along τ_q given by

$$Y_q(s) = \mathcal{M}_{a_1,\rho}(s)\frac{\partial}{\partial\theta}|_{\tau_q}, \quad s \in [\rho, \infty),$$

where $\mathcal{M}_{a_1,\rho}(s)$ is a smooth function along $\tau_q|_{[\rho,\infty)}$ depending on a constant a_1 chosen such that m' is positive on $[0, a_1]$ and ρ.

Moreover, if consider the vector field $J(s)$, along the twisted meridian $\mathcal{R}_q : [\rho, \infty) \to M$, $\mathcal{R}_q(s) = \varphi(s, \tau_q(s))$, defined by

$$J(s) := \varphi_{\tau_q,*}(Y_q(s)),$$

then one can see that J is actually a Jacobi field along $\mathcal{R}_q$. Indeed, one can easily verify that the flow φ of W maps the solutions of the Jacobi equation for Y_q into the solutions of the Jacobi equation for $J(s)$, and therefore we have proved that the first F-conjugate point of q is obtained at the intersection of the parallel through the first h-conjugate point with τ_q.

At the second step, we will do the same thing for cut points of q, i.e. we will establish the correspondence of h-cut points of q with the F-cut points of q. Namely, we will show that a point $\tilde{y} \in \tau_q|_{[c,\infty)}$ is an h-cut point of q if and only if the point y, found at the intersection of the parallel through $\tilde{y}$ with the twisted meridian $\{\varphi(s, \tau_q(s)) : s \in [c, \infty)\}$ is an F-cut point of q.

Indeed, such a $\tilde{y}$ is an h-cut point of q if and only if there exists two h-geodesic segments α_1 and α_2 on M from q to $\tilde{y}$ of equal h-length. By making use of Theorem 1.1 and an argument similar to Proposition 3, we can see that under the action of the flow φ the end point $\tilde{y}$ is clearly mapped into the point y described above and the h-maximal geodesic segments α_1 and α_2 are deviated into two F-geodesic segments of same F-length from q to y. This concludes the proof (see Figure 1).

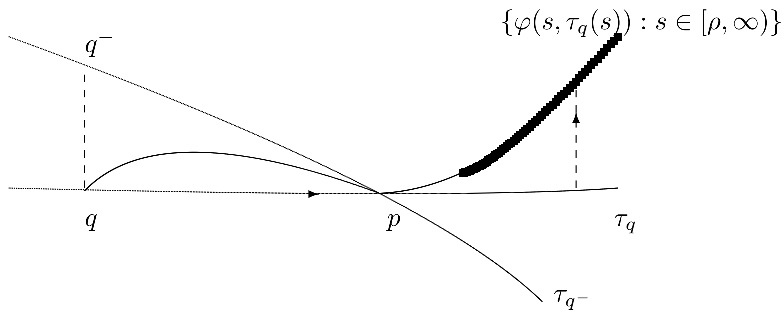

Figure 1. The thick line is the F-cut locus of q.

Here are other two small typos on p. 481, line 2 from bottom. There is a "2" missing in both formulas. The correct formulas are:

$$(L)^+_F(r_0) = \frac{2\pi m(r_0)}{1+\mu\cdot m(r_0)} \quad \text{and} \quad (L)^-_F(r_0) = \frac{2\pi m(r_0)}{1-\mu\cdot m(r_0)}\,.$$

On p. 486, line 10 from top, the formula

$$\frac{\partial\alpha^2}{\partial y^2} = 2\alpha_{22}y^2 \quad \text{should be} \quad \frac{\partial\alpha^2}{\partial y^2} = 2a_{22}y^2\,.$$

References

[1] R. Hama, P. Chitsakul and S. V. Sabau, The geometry of a Randers rotational surface, *Publ. Math. Debrecen* **87** (2015), 473–502.

[2] K. Shiohama, T. Shioya and M. Tanaka, The Geometry of Total Curvature on Complete Open Surfaces, Cambridge Tracts in Mathematics **159**, *Cambridge University Press, Cambridge*, 2003.

[3] M. Tanaka, On the cut loci of a von Mangoldt's surface of revolution, *J. Math. Soc. Japan* **44** (1992), 631–641.

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(Received October 26, 2015)