Title: On the variety of bands in completely regular semigroups

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Completely regular semigroups, enriched by the unary operation of inversion within their maximal subgroups, form a variety $\mathcal{CR}$ whose lattice of subvarieties is denoted by $\mathcal{L}(\mathcal{CR})$. Its subvariety $\mathcal{B}$ of all bands plays a seminal role in any study of the structure of $\mathcal{L}(\mathcal{CR})$. We present some new aspects of $\mathcal{B}$ relative to both $\mathcal{CR}$ and the variety $\mathcal{CS}$ of completely simple semigroups.

Since $\mathcal{B}$ is neutral in $\mathcal{L}(\mathcal{CR})$, the latter is a subdirect product of the lattice $(\mathcal{B}]$ of subvarieties of $\mathcal{B}$ and the lattice $[\mathcal{B})$ of supervarieties of $\mathcal{B}$. We determine the precise image of $\mathcal{L}(\mathcal{CR})$ in $(\mathcal{B}] \times [\mathcal{B})$.

For the relation $\mathbf{B}^{\vee}$ defined on $\mathcal{L}(\mathcal{CR})$ by $\mathcal{U}\mathbf{B}^{\vee}\mathcal{V}$ if $\mathcal{U} \vee \mathcal{B} = \mathcal{V} \vee \mathcal{B}$, we prove that each $\mathbf{B}^{\vee}$-class is embeddable into $(\mathcal{B}]$.

We establish several results concerning the variety $\mathcal{CS}$ in the context of the relation $\mathbf{B}^{\vee}$ and the structure of the lattice $\mathcal{L}(\mathcal{CS})$.

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