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Title: Approximately Jensen-convex functions

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In this paper we show that if a function satisfies the Jensen-inequality (or the inequality describing $\mathbb{Q}$-convexity) with an appropriate error term, then the function is Jensen-convex (without error) as well.

First we consider a function f, which is defined on an open interval I of $\mathbb{R}$. We prove that if $f : I \to \mathbb{R}$ satisfies the inequality

$$f\left(\frac{x+y}{2}\right) \leq \frac{f(x)+f(y)}{2} + \psi(|x-y|)$$

for every $x, y \in I$, where $\lim\limits_{t\to 0+} \frac{\psi(t)}{t^2} = 0$, then f is Jensen-convex.

We also prove that if a real function f, which is defined on an $\mathbb{F}$-algebraically open and $\mathbb{F}$-convex subset D of a vector space X over $\mathbb{F}$ (where $\mathbb{F}$ is a subfield of $\mathbb{R}$), satisfies the inequality

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y) + c\,[\lambda(1-\lambda)\,|x-y|]^p$$

for every $x, y \in D$ and $\lambda \in [0,1] \cap \mathbb{F}$, with a fixed non-negative real number c and a fixed exponent $p > 1$, then it has to be $\mathbb{F}$-convex, i.e., f satisfies the above inequality with $c = 0$ as well. Considering $\mathbb{F} = \mathbb{Q}$, we obtain another characterization of Jensen-convex functions.

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