Title: Some geometrical properties of four-dimensional Lorentzian Damek–Ricci spaces

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In this paper, we investigate some geometrical properties of four-dimensional Lorentzian Damek–Ricci spaces, including some problems related to Ricci solitons, harmonicity of invariant vector fields and curvature properties. We show that these spaces does not even admit a left-invariant Ricci soliton, although all Riemannian Damek–Ricci spaces are Einstein manifolds. Besides, we determine all the vector fields which are critical points for the energy functional restricted to vector fields of the same length. We also prove that there does not exist any invariant harmonic vector field or invariant vector field which defines a harmonic map. Finally, we determine all the invariant unit time-like vector fields which are spatially harmonic.

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