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Some results concerning symmetric generalized skew biderivations on prime rings

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Abstract. Let R be a ring. A biadditive symmetric mapping $D : R \times R \longrightarrow R$ is called a symmetric skew biderivation if for every $x \in R$, the map $y \mapsto D(x, y)$ is a skew derivation of R (as well as for every $y \in R$, the map $x \mapsto D(x, y)$ is a skew derivation of R).

Let $D : R \times R \longrightarrow R$ be a symmetric biderivation. A biadditive symmetric mapping $\Delta : R \times R \longrightarrow R$ is said to be a symmetric generalized skew biderivation if for every $x \in R$, the map $y \mapsto \Delta(x, y)$ is a generalized skew derivation of R associated with D (as well as for every $y \in R$, the map $x \mapsto \Delta(x, y)$ is a generalized skew derivation of R associated with D).

In this paper we study some commutativity conditions for a prime ring R related to the behaviour of the trace of symmetric generalized skew biderivations of R.

1. Introduction

Throughout, R will be a prime ring with center $Z(R)$. We denote the *right Martindale quotient ring* of R by Q. The center of Q is denoted by C, which is called *extended centroid* of R. R is a prime ring if and only if C is a field. We refer the reader to [3] for more details.

An additive mapping $d \colon R \longrightarrow R$ is said to be a *derivation* of R if

$$d(xy) = d(x)y + xd(y)$$

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for all $x, y \in R$. An additive mapping $F\colon R \longrightarrow R$ is called a *generalized derivation* of R if there exists a derivation d of R such that

$$F(xy) = F(x)y + xd(y)$$

for all $x, y \in R$. The derivation d is uniquely determined by F, which is called an *associated derivation* of F.

The definition of generalized skew derivation is a unified notion of skew derivation and generalized derivation, which are considered as classical additive mappings of non-commutative algebras. Let R be an associative ring and α be an automorphism of R. An additive mapping $d\colon R \longrightarrow R$ is said to be a *skew derivation* of R if

$$d(xy) = d(x)y + \alpha(x)d(y)$$

for all $x, y \in R$. The automorphisms α is called an *associated automorphism* of d. An additive mapping $F\colon R \longrightarrow R$ is called a *generalized skew derivation* of R if there exists a skew derivation d of R with associated automorphism α such that

$$F(xy) = F(x)y + \alpha(x)d(y)$$

for all $x, y \in R$. In this case, d is called an *associated skew derivation* of F and α is called an *associated automorphism* of F.

In this paper we will study the structure of two appropriate maps $\delta_1, \delta_2 : R \to R$, satisfying the condition $\delta_1(x)\delta_2(x) = 0$, for any $x \in R$. In literature, several answers to the above-mentioned problem exist. In particular, many authors study the case when δ_1 and δ_2 are additive maps of R, such as derivations, generalized derivations and skew derivations. For instance, in [2] it is proved that if R is a prime ring with infinite extended centroid and $\delta_1, \dots, \delta_n$ are derivations of R such that $\delta_1(x)\delta_2(x)\cdots\delta_n(x) = 0$, for all $x \in R$, then at least one δ_i is trivial.

Later, in [14], Vukman extends the previous result to skew derivations, in the case $n = 2$. More precisely, let $\delta_1, \delta_2 : R \to R$ be skew derivations with associated automorphism α. If $\delta_1(x)\delta_2(x) = 0$, for all $x \in R$, then either $\delta_1 = 0$ or $\delta_2 = 0$.

Recently, in [15] this result has been generalized to the case of (α, β)-derivations. We recall that an additive $d : R \to R$ is called (α, β)-derivation if $d(xy) = d(x)\alpha(y) + \beta(x)d(y)$, for all $x, y \in R$ and for fixed automorphisms α, β of R. In [15], it is proved that if δ_1 and δ_2 are (α, β)-derivations such that either δ_1 or δ_2 commutes with α and β, and $\delta_1(x)\delta_2(x) = 0$, for all $x \in R$, then either $\delta_1 = 0$ or $\delta_2 = 0$.

More recently, M. Fošner and Vukman have considered an analogous problem, where derivations and (α,β)-derivations are replaced by generalized derivations. In Theorem 3 in [7], they prove that if δ_1 and δ_2 are generalized derivations of a prime ring R of characteristic different from 2, such that $\delta_1(x)\delta_2(x)=0$, for all $x\in R$, then there exist p,q elements of the Martindale quotient ring Q of R, such that $\delta_1(x)=xp$ and $\delta_2(x)=qx$ for all $x\in R$ and $pq=0$, except when at least one δ_i is zero.

In [13], Vukman considers a different approach, by studying similar problems when the maps $\delta_1,\delta_2 : R\to R$ are not additive. More precisely, he analyzes the case where δ_1,δ_2 are traces of symmetric biderivations and proves that if R is a prime ring of characteristic different from two and three, D_1,D_2 are two symmetric biderivations of R, δ_1,δ_2 are the traces of D_1 and D_2, respectively, such that $\delta_1(x)\delta_2(x)=0$ for any $x\in R$, then either $D_1=0$ or $D_2=0$.

Following this line of investigation, we extend the result in [13] to symmetric generalized skew biderivations and prove the following results:

Theorem 1. *Let R be a prime ring of characteristic different from two and three, D a symmetric skew biderivation of R, associated with the automorphism α of R, $\Delta : R\times R\longrightarrow R$ be the symmetric generalized skew biderivation associated with α and D, and let δ be the trace of Δ. If $x\delta(x)=0$ for any $x\in R$, then $\Delta=0$.*

Theorem 2. *Let R be a prime ring of characteristic different from two and three, D a symmetric skew biderivation of R, associated with the automorphism α of R, $\Delta : R\times R\longrightarrow R$ be the symmetric generalized skew biderivation associated with α and D, and let δ be the trace of Δ. If $\alpha(x)\delta(x)=0$ for any $x\in R$, then $\Delta=0$.*

Theorem 3. *Let R be a prime ring of characteristic different from two and three, D_1,D_2 two symmetric skew biderivations of R, associated with the automorphism α of R, $\Delta_1,\Delta_2 : R\times R\longrightarrow R$ two symmetric generalized skew biderivations associated with α and respectively with D_1 and D_2, and let δ_1,δ_2 be the traces respectively of Δ_1 and Δ_2. If $\delta_1(x)\delta_2(x)=0$ for any $x\in R$, then either $\Delta_1=0$ or $\Delta_2=0$.*

2. Preliminaries

Let $D : R\times R\longrightarrow R$ be a biadditive map. We say that D is symmetric if $D(x,y)=D(y,x)$, for all $x,y\in R$. A mapping $f : R\longrightarrow R$ defined by

$f(x) = D(x,x)$, where $D : R \times R \longrightarrow R$ is a symmetric mapping, is called the trace of D. It is obvious that in the case $D : R \times R \longrightarrow R$ is a symmetric mapping which is also biadditive (i.e. additive in both arguments), the trace f of D satisfies the relation $f(x+y) = f(x) + f(y) + 2D(x,y)$, for all $x, y \in R$. A biadditive mapping $D : R \times R \longrightarrow R$ is called a biderivation if $D(xy,z) = D(x,z)y + xD(y,z)$ for all $x, y, z \in R$. Obviously, in this case the relation $D(x,yz) = D(x,y)z + yD(x,z)$ is also satisfied for all $x, y, z \in R$.

In [10], Maksa introduces the concept of a symmetric biderivation (see also [11]), where an example can be found). It was shown in [10] that symmetric biderivations are related to the general solution of some functional equations.

The notion of generalized biderivation is introduced by Argaç in [1].

Let $D : R \times R \longrightarrow R$ be a biderivation. A biadditive mapping $\Delta : R \times R \longrightarrow R$ is said to be a generalized biderivation if for every $x \in R$, the map $y \mapsto \Delta(x,y)$ is a generalized derivation of R associated with D as well as for every $y \in R$, the map $x \mapsto \Delta(x,y)$ is a generalized derivation of R associated with D, i.e., $\Delta(x,yz) = \Delta(x,y)z + yD(x,z)$ and $\Delta(xy,z) = \Delta(x,z)y + xD(y,z)$ for all $x, y, z \in R$.

Let $D : R \times R \longrightarrow R$ be a symmetric biadditive mapping, α an automorphism of R. D is said to be a symmetric skew biderivation associated with α if for every $x \in R$, the map $y \mapsto D(x,y)$ is a skew derivation of R associated with α as well as for every $y \in R$, the map $x \mapsto D(x,y)$ is a skew derivation of R associated with α, i.e., $D(x,yz) = D(x,y)z + \alpha(y)D(x,z)$ and $D(xy,z) = D(x,z)y + \alpha(x)D(y,z)$ for all $x, y, z \in R$.

Let $D : R \times R \longrightarrow R$ be a symmetric skew biderivation of R, associated with the automorphism α of R. The symmetric biadditive mapping $\Delta : R \times R \longrightarrow R$ is said to be a symmetric generalized skew biderivation associated with α and D, if for every $x \in R$, the map $y \mapsto \Delta(x,y)$ is a generalized skew derivation of R associated with α and D, as well as for every $y \in R$, the map $x \mapsto \Delta(x,y)$ is a generalized skew derivation of R associated with α and D, i.e., $\Delta(x,yz) = \Delta(x,y)z + \alpha(y)D(x,z)$ and $\Delta(xy,z) = \Delta(x,z)y + \alpha(x)D(y,z)$ for all $x, y, z \in R$.

In order to prove our results, we need to recall the following known Facts:

Fact 1. In [4], Chang extends the definition of generalized skew derivation to the right Martindale quotient ring Q of R as follows: by a (right) generalized skew derivation we mean an additive mapping $G : Q \longrightarrow Q$ such that $G(xy) = G(x)y + \alpha(x)d(y)$ for all $x, y \in Q$, where d is a skew derivation of R and α is an automorphism of R. Moreover, there exists $G(1) = a \in Q$ such that $G(x) = ax + d(x)$ for all $x \in R$.

Fact 2. In [5], Chuang and Lee investigate polynomial identities with skew derivations. They prove that if $\Phi(x_i, D(x_i))$ is a generalized polynomial identity for R, where R is a prime ring and D in an outer skew derivation of R, then R also satisfies the generalized polynomial identity $\Phi(x_i, y_i)$, where x_i and y_i are distinct indeterminates. Furthermore, they prove [5, Theorem 1] that in the case $\Phi(x_i, D(x_i), \alpha(x_i))$ is a generalized polynomial identity for R, where R is a prime ring, D is an outer skew derivation of R and α is an outer automorphism of R, then R also satisfies the generalized polynomial identity $\Phi(x_i, y_i, z_i)$, where x_i, y_i, and z_i are distinct indeterminates.

Fact 3. By [5, Theorem 1] we have the next result. If d is a non-zero skew-derivation of R and

$$\Phi\Big(x_1, \dots, x_n, d(x_1), \dots, d(x_n)\Big)$$

is a skew-differential identity of R, then one of the following statements hold:

(1) either d is inner;

(2) or R satisfies the generalized polynomial identity

$$\Phi(x_1, \dots, x_n, y_1, \dots, y_n).$$

3. The proof of Theorem 1

Lemma 1. *Let R be a prime ring of characteristic different from two, D a symmetric skew biderivation of R, associated with the automorphism α of R, $\Delta : R \times R \longrightarrow R$ be the symmetric generalized skew biderivation associated with α and D and δ the trace of Δ. If $x\delta(x) = 0$ for any $x \in R$, then $y^2\Delta(x, y) - y\alpha(y)D(x, y) = 0$, for all $x, y \in R$.*

Proof. We start from $x\delta(x) = 0$ for any $x \in R$, that is

$$x\Delta(x, x) = 0, \quad \forall x \in R. \tag{1}$$

The linearization of (1) gives us

$$\begin{aligned} &x\Delta(x, y) + x\Delta(y, x) + x\Delta(y, y) \\ &\quad + y\Delta(x, x) + y\Delta(x, y) + y\Delta(y, x) = 0, \quad \forall x, y \in R. \end{aligned} \tag{2}$$

Substituting $-x$ for x in (2), we have

$$\begin{aligned} &x\Delta(x, y) + x\Delta(y, x) - x\Delta(y, y) \\ &\quad + y\Delta(x, x) - y\Delta(x, y) - y\Delta(y, x) = 0, \quad \forall x, y \in R, \end{aligned} \tag{3}$$

and from (2) and (3) it follows that

$$x\Delta(y,y)+2y\Delta(x,y)=0,\quad \forall x,y\in R. \tag{4}$$

Replacing x by yx in (4), we get

$$yx\Delta(y,y)+2y\Delta(y,y)x+2y\alpha(y)D(x,y)=0,\quad \forall x,y\in R. \tag{5}$$

Using (1) and (4) in (5), one has

$$y^2\Delta(x,y)-y\alpha(y)D(x,y)=0,\quad \forall x,y\in R. \tag{6}$$

□

Lemma 2. *Let R be a prime ring of characteristic different from two, D a symmetric biderivation of R and δ the trace of D. If $x\delta(x)=0$ for any $x\in R$, then $D=0$.*

PROOF. Firstly, we consider the case when R is commutative. Then, by our hypothesis, for any $0\neq x\in R$ it follows $\delta(x)=0$, and by linearizing this relation we get $D(x,y)=0$ for any $x,y\in R$.

Therefore, in all that follows we assume that R is not commutative.

Replacing x with xy in (4) and using (1), we get $2yD(xy,y)=0$, that is

$$yD(x,y)y+yxD(y,y)=0,\quad \forall x,y\in R. \tag{7}$$

Using (4), in (7) it follows that

$$yD(x,y)y-2y^2D(x,y)=0,\quad \forall x,y\in R. \tag{8}$$

Hence, for any $x_0\in R$, the map $F(y)=D(x_0,y)$, for all $y\in R$, is a derivation of R such that

$$yF(y)y-2y^2F(y)=0,\quad \forall y\in R. \tag{9}$$

If $F=0$ for any $x_0\in R$, then $D(x,y)=0$ for all $x,y\in R$ and we are done. Let $x_0\in R$ be such that the related F is not zero. We prove that in this case a contradiction follows.

By KHARCHENKO's theorem in [9], if F is outer, we have the contradiction $yty-2y^2t=0$ for all $y,t\in R$. Then we may assume that there exists an element $p\in Q$ such that $F(y)=[p,y]$, for any $y\in R$, moreover, $p\notin C$. Thus $y[q,y]y-2y^2[q,y]$ is a non-trivial generalized polynomial identity for R. By [12],

Q is a primitive ring having nonzero socle with the field C as its associated division ring. By [8, p. 75] Q is isomorphic to a dense subring of the ring of linear transformations of a vector space V over C, containing nonzero linear transformations of finite rank. Since R is not commutative, $\dim_C V \geq 2$. Moreover, since $p \notin C$, there exists $v \in V$ such that $\{v, pv\}$ is not linearly dependent over C. By the density of R, there exists $r \in R$ such that $rv = 0$ and $rpv = pv$. Therefore, the following contradiction occurs:

$$0 = (r[q,r]r - 2r^2[q,r])v = 2pv \neq 0. \qquad \square$$

Lemma 3. *Let R be a prime ring of characteristic different from two, D a symmetric biderivation of R, Δ a symmetric generalized biderivation of R, associated with the symmetric biderivation D, and δ the trace of Δ. If $x\delta(x) = 0$ for any $x \in R$, then $\Delta = 0$.*

Proof. For any $x_0 \in R$, we consider again the following additive maps on R:

$$F(y) = \Delta(x_0, y),\ \forall y \in R \quad \text{and} \quad f(y) = D(x_0, y),\ \forall y \in R.$$

Here F is a generalized derivation of R with associated derivation f. Moreover, by (6) we have that

$$y^2F(y) - y^2f(y) = 0, \quad \forall y \in R. \tag{10}$$

Since there exists $a \in Q$ such that $F(y) = ay + f(y)$, for all $y \in R$, then $y^2ay = 0$ and $a = 0$ follows. Therefore, $\Delta(x_0, y) = D(x_0, y)$, for all $y \in R$. Repeating this process for any $x_0 \in R$, it follows that $\Delta(x, y) = D(x, y)$, for all $x, y \in R$, that is Δ is a symmetric biderivation of R. Thus the conclusion follows from Lemma 2. $\square$

Proof of Theorem 1. By the same argument as in Lemma 2, we may assume that R is not commutative.

We start from relation (6), that is

$$y^2\Delta(x, y) - y\alpha(y)D(x, y) = 0, \quad \forall x, y \in R.$$

Again we fix $x_0 \in R$ and introduce the following additive maps on R:

$$F(y) = \Delta(x_0, y),\ \forall y \in R \quad \text{and} \quad f(y) = D(x_0, y),\ \forall y \in R.$$

Notice that F is a generalized skew derivation of R with associated automorphism α and associated skew derivation f. Moreover, by (6) we have that

$$y^2F(y) - y\alpha(y)f(y) = 0, \quad \forall y \in R. \tag{11}$$

By Fact 1, there exists $a \in Q$ such that $F(y) = ay + f(y)$, for all $y \in R$, so that

$$y^2(ay + f(y)) - y\alpha(y)f(y) = 0, \quad \forall y \in R. \tag{12}$$

Note that if $f = 0$, then $y^2 ay = 0$ and $a = 0$ follow: in this case $F = 0$.

Consider $f \neq 0$. If f is an outer skew derivation of R, then by (12) it follows that R satisfies

$$y^2(ay + t) - y\alpha(y)t = 0, \quad \forall y \in R, \tag{13}$$

in particular, $y^2 t - y\alpha(y)t = 0$, and by the primeness of R we get $y^2 - y\alpha(y) = 0$, for all $y \in R$.

In case α is an outer automorphism of R, then we get the contradiction $y^2 - yt = 0$ for any $y, t \in R$. Therefore, there exists an invertible element $q \in Q$ such that $\alpha(y) = qyq^{-1}$ and

$$y^2 - yqyq^{-1} = 0, \quad \forall y \in R. \tag{14}$$

Firstly, we assume that $q \notin C = Z(Q)$, so that (14) is a non-trivial generalized polynomial identity for R. By [12], Q is a primitive ring having nonzero socle with the field C as its associated division ring. By [8, p. 75], Q is isomorphic to a dense subring of the ring of linear transformations of a vector space V over C, containing nonzero linear transformations of finite rank. Since R is not commutative, $\dim_C V \geq 2$, so that Q contains some non-trivial idempotent elements, say $e^2 = e \in Q$. For any $x \in R$, replacing y by $ex(1-e)$ in (14,), we get $ex(1-e)qex(1-e)q^{-1} = 0$, that is $eq(1-e) = 0$. Thus the contradiction $q \in C$ follows.

The previous argument says that if f is outer, then α is the identity map.

Let now f be inner, that is there exists $b \in Q$ such that $f(x) = bx - \alpha(x)b$, for all $x \in R$. Hence (12) reduces to

$$y^2(ay + by - \alpha(y)b) - y\alpha(y)(by - \alpha(y)b) = 0, \quad \forall y \in R. \tag{15}$$

If α is outer, then (15) implies that

$$y^2(ay + by - tb) - yt(by - tb) = 0, \quad \forall y, t \in R. \tag{16}$$

As above, since R satisfies a generalized polynomial identity, Q is a primitive ring and it is isomorphic to a dense subring of the ring of linear transformations of a vector space V over C, containing nonzero linear transformations of finite rank. As above $\dim_C V \geq 2$, then Q contains some non-trivial idempotent elements, say $e^2 = e \in Q$. Replacing y by $ex(1-e)$ in (16) and right multiplying by e, we get

$ex(1-e)t^2be = 0$, for all $x, t \in R$, that is $be = 0$. Analogously, for $y = (1-e)xe$ in (16), we have $b(1-e) = 0$, therefore, $b = 0$. Thus, by (16) it follows $y^2ay = 0$ for all $y \in R$, i.e. $a = 0$ and $F = 0$.

Finally, we consider the case when there exists an invertible element $q \in Q$ such that $\alpha(y) = qyq^{-1}$ and

$$y^2(ay + by - qyq^{-1}b) - yqyq^{-1}(by - qyq^{-1}b) = 0, \quad \forall y \in R. \tag{17}$$

If $q^{-1}b \in C$, then $f = 0$ and as above $a = 0$ and $F = 0$. Moreover, if $q \in C$, then α is the identity map. We consider here both $q^{-1}b \notin C$ and $q \notin C$ and prove that a contradiction follows.

Once again R satisfies a non-trivial generalized polynomial identity. As above, we may assume that Q is isomorphic to a dense subring of the ring of linear transformations of a vector space V over C, with $\dim_C V \geq 2$.

If $\dim_C V = k$ is finite, then $Q = M_k(C)$, the ring of $k \times k$ matrices over C, with $k \geq 2$. Assume firstly that C is infinite. Since both $q^{-1}b \notin C$ and $q \notin C$, then, by [6, Lemma 1.5], there exists an invertible matrix $P \in M_k(C)$ such that each matrix $u = Pq^{-1}bP^{-1}$ and $v = PqP^{-1}$ has all non-zero entries. Moreover, by (17) it is easy to see that

$$y^2(ay + by - vyu) - yvyuy + yvy^2u = 0, \quad \forall y \in R. \tag{18}$$

Let $y = e_{ij}$ for any $i \neq j$, the usual matrix unit with 1 in the (i, j)-entry and zero elsewhere, then $e_{ij}ve_{ij}ue_{ij} = 0$, which is a contradiction.

Now let E be an infinite field which is an extension of the field C and let $\overline{R} = M_t(E) \cong R \otimes_C E$. Consider the generalized polynomial

$$\Psi(y) = y^2(ay + by - qyq^{-1}b) - yqyq^{-1}(by - qyq^{-1}b),$$

which is a generalized polynomial identity for R. Moreover, it is homogeneous of degree 3 in the indeterminate y. Hence, the complete linearization of $\Psi(y)$ is a multilinear generalized polynomial $\Theta(y, z)$. Moreover, $\Theta(y, y) = 3\Psi(y)$. Clearly, the multilinear polynomial $\Theta(y, y)$ is a generalized polynomial identity for R and $\overline{R}$ too. Since $\mathrm{char}(C) \neq 3$, we obtain $\Psi(r) = 0$ for all $r \in \overline{R}$, and the conclusion follows from the above argument.

Let now $\dim_C V = \infty$. Recall that if an element $r \in R$ centralizes the non-zero ideal $H = \mathrm{soc}(RC)$, then $r \in C$.

Hence, we may assume there exist $r_1, r_2 \in H = \mathrm{soc}(RC)$ such that $[q, r_1] \neq 0$ and $[q^{-1}b, r_2] \neq 0$, and prove that a number of contradictions follows.

By Litoff's Theorem [8, Page 90], there exists $e^2 = e \in H$ such that

- $r_1, r_2 \in eRe$;
- $ar_1, r_1a, ar_2, r_2a \in eRe$;
- $br_1, r_1b, br_2, r_2b \in eRe$;
- $qr_1, r_1q, qr_2, r_2q \in eRe$;
- $q^{-1}br_1, r_1q^{-1}b, q^{-1}br_2, r_2q^{-1}b \in eRe$;

where $eRe \cong M_m(C)$, the matrix ring over the extended centroid C. Note that eRe satisfies (17). By the above matrix case, we have that one of the following assertions holds:

(1) $eqe \in C$, which contradicts with the choice of $r_1 \in H$;

(2) $eq^{-1}be \in C$, which contradicts with the choice of $r_2 \in H$.

All the previous arguments imply that either $F(y) = \Delta(x_0, y) = 0$ for all $y \in R$, or α is the identity map. Repeating this process for all $x_0 \in R$, it follows that $\Delta(x, y) = 0$ for all $x, y \in R$, unless α is the identity map.

In the latter case, Δ is a generalized symmetric biderivation associated with the symmetric biderivation D. Thus, by Lemma 3 we are done.

4. The proof of Theorem 2

Lemma 4. *Let R be a prime ring of characteristic different from two, D a symmetric skew biderivation of R, associated with the automorphism α of R, $\Delta : R \times R \longrightarrow R$ be the symmetric generalized skew biderivation associated with α and D and δ the trace of Δ. If $\alpha(x)\delta(x) = 0$ for any $x \in R$, then $\alpha(x)\delta(y) + 2\alpha(y)\Delta(x, y) = 0$, for all $x, y \in R$.*

Proof. We start from $\alpha(x)\delta(x) = 0$ for any $x \in R$, that is

$$\alpha(x)\Delta(x, x) = 0, \quad \forall x \in R. \tag{19}$$

The linearization of (19) gives us

$$\alpha(x)\delta(y) + 2\alpha(x)\Delta(x, y) + \alpha(y)\delta(x) + 2\alpha(y)\Delta(x, y) = 0, \quad \forall x, y \in R. \tag{20}$$

Substituting $-x$ for x in (20) we have

$$\alpha(x)\delta(y) + 2\alpha(y)\Delta(x, y) = 0, \quad \forall x, y \in R. \tag{21}$$

□

Lemma 5. *Let R be a prime ring of characteristic different from two, D a symmetric skew biderivation of R, associated with the automorphism α of R, and δ the trace of D. If $\alpha(x)\delta(x) = 0$ for any $x \in R$, then $D = 0$.*

Proof. If R is commutative, then for any $0 \neq x \in R$ it follows that $\delta(x) = 0$, and linearizing this relation we get $D(x, y) = 0$ for any $x, y \in R$. Thus we may assume in the sequel that R is not commutative.

We fix $x_0 \in R$, by (21) we have that

$$\alpha(y)\delta(x_0) + 2\alpha(x_0)\Delta(y, x_0) = 0, \quad \forall y \in R. \tag{22}$$

Here we denote $b = \delta(x_0)$, $c = 2\alpha(x_0)$ and introduce the following additive map on R:

$$f(y) = D(y, x_0), \quad \forall y \in R.$$

Notice that f is a skew derivation of R with associated automorphism α. Therefore, by (22)

$$\alpha(y)b + cf(y) = 0, \quad \forall y \in R. \tag{23}$$

If $f = 0$, then $b = 0$ follows from (23). Assume that $f \neq 0$ is outer, then by (23) it follows that $\alpha(y)b + cz = 0$, for all $y, z \in R$. In particular $\alpha(y)b = 0$, for any $y \in R$, that is $b = 0$ again. Let now $f(x) = vx - \alpha(x)v$ for any $x \in R$ and for a fixed element $0 \neq v \in Q$. Hence R satisfies

$$\alpha(y)b + cvy - c\alpha(y)v = 0, \quad \forall y \in R. \tag{24}$$

If α is outer, then (24) implies that R satisfies $zb + cvy - czv = 0$. In particular $cvy = 0$ for any $y \in R$, i.e. $cv = 0$. Thus $zb - czv = 0$, for any $z \in R$. Replacing z with vz, we get $vzb = 0$ for all $z \in R$, and since $v \neq 0$ it follows that $b = 0$.

Consider now the case when there exists an invertible element $q \in Q$ such that $\alpha(y) = qyq^{-1}$ and

$$qyq^{-1}b + cvy - cqyq^{-1}v = 0, \quad \forall y \in R. \tag{25}$$

Notice that since $f \neq 0$, $q^{-1}v \notin C$. Moreover, if $c \in C$, then $\alpha(x_0) \in C$, and by our main assumption it follows that $b = \delta(x_0) = 0$. Thus, we may also assume that $c \notin C$. In light of this, (25) is a non-trivial generalized polynomial identity for R. As above, we may assume that Q is isomorphic to a dense subring of the ring of linear transformations of a vector space V over C, with $\dim_C V =\geq 2$.

If $\dim_C V = k$ is finite, then $Q = M_k(C)$, the ring of $k \times k$ matrices over C, with $k \geq 2$. Assume firstly that C is infinite. Left multiplying (25) by q^{-1}, we have that

$$yq^{-1}b + q^{-1}cvy - q^{-1}cqyq^{-1}v = 0, \quad \forall y \in R. \tag{26}$$

Since both $q^{-1}v \notin C$ and $q^{-1}cq \notin C$, by [6, Lemma 1.5], there exists an invertible matrix $P \in M_k(C)$ such that each matrix $u = Pq^{-1}vP^{-1}$ and $w = Pq^{-1}cqP^{-1}$ has all non-zero entries. Denote $\overline{b} = Pq^{-1}bP^{-1}$, $\overline{v} = Pq^{-1}cvP^{-1}$, then by (26) it is easy to see that

$$y\overline{b} + \overline{v}y - wyu = 0, \quad \forall y \in R. \tag{27}$$

For $y = e_{ij}$ $(i \neq j)$ in relation (27), both left and right multiplying by e_{ij}, we get $e_{ij}we_{ij}ue_{ij} = 0$, which is a contradiction, since u and w have all non-zero entries.

Now let E be an infinite field which is an extension of the field C and let $\overline{R} = M_t(E) \cong R \otimes_C E$. Consider the generalized polynomial

$$\Psi(y) = yq^{-1}b + q^{-1}cvy - q^{-1}cqyq^{-1}v$$

which is a generalized polynomial identity for R. Moreover, it is a linear identity in the indeterminate y. Hence $\Psi(y)$ is a generalized polynomial identity for $\overline{R}$ too, and the conclusion follows from the above argument.

Let now $\dim_C V = \infty$. We may assume there exist $r_1, r_2 \in H = \mathrm{soc}(RC)$ such that $[c, r_1] \neq 0$ and $[q^{-1}v, r_2] \neq 0$ and prove that a number of contradictions follows.

By Litoff's Theorem [8, Page 90], there exists $e^2 = e \in H$ such that

- $r_1, r_2 \in eRe$;
- $cr_1, r_1c, cr_2, r_2c \in eRe$;
- $q^{-1}br_1, r_1q^{-1}b, q^{-1}br_2, r_2q^{-1}b \in eRe$;
- $q^{-1}vr_1, r_1q^{-1}v, q^{-1}vr_2, r_2q^{-1}v \in eRe$;
- $q^{-1}cqr_1, r_1q^{-1}cq, q^{-1}cqr_2, r_2q^{-1}cq \in eRe$;
- $q^{-1}cvr_1, r_1q^{-1}cv, q^{-1}cvr_2, r_2q^{-1}cv \in eRe$;

where $eRe \cong M_m(C)$, the matrix ring over the extended centroid C. Note that eRe satisfies (26). By the above matrix case, we have that one of the following assertions hold:

(1) $ece \in C$, which contradicts with the choice of $r_1 \in H$;

(2) $eq^{-1}ve \in C$, which contradicts with the choice of $r_2 \in H$.

All the previous arguments imply that $b = \delta(x_0) = 0$. Repeating this process for all $x_0 \in R$, it follows that $D(x, x) = 0$ for all $x \in R$. Finally, by linearizing this relation, we have that $D(x, y) = 0$, for all $x, y \in R$. □

Proof of Theorem 2. Replacing x with yx in relation (21) and using again (21), it follows

$$\alpha(y^2)\Big(\Delta(x, y) - D(x, y)\Big) = 0, \quad \forall x, y \in R. \tag{28}$$

As above, we fix $x_0 \in R$ and introduce the following additive maps on R:

$$F(y) = \Delta(x_0, y),\ \forall y \in R \quad \text{and} \quad f(y) = D(x_0, y),\ \forall y \in R.$$

Notice that F is a generalized skew derivation of R with associated automorphism α and associated skew derivation f. Moreover, by (28) we have that

$$\alpha(y^2)\Big(F(y) - f(y)\Big) = 0, \quad \forall y \in R. \tag{29}$$

By Fact 1, there exists $a \in Q$ such that $F(x) = ax + f(x)$, for all $x \in R$, so that $\alpha(y^2)ay = 0$, for all $y \in R$. Easy computations show that in this case $a = 0$, that is $F = f$, in other words $\Delta(x_0, y) = D(x_0, y)$ for any $y \in R$. Repeating this process for all $x_0 \in R$, it follows that $\Delta(x, y) = D(x, y)$ for any $x, y \in R$, then Δ is symmetric skew biderivation of R and the conclusion follows from Lemma 5.

5. The proof of Theorem 3

We premit the following easy result on symmetric biadditive maps, which will be useful in the sequel. Notice that in the next Lemma, the hypothesis on the primeness of the ring R is not needed. Moreover, the involved symmetric biadditive maps are not requested to be neither biderivations, nor generalized biderivations, nor generalized skew biderivations:

Lemma 6. *Let R be a ring of characteristic different from two and three, $\Delta_1, \Delta_2 : R \times R \longrightarrow R$ two symmetric biadditive maps of R, δ_1 the trace of Δ_1, and δ_2 the trace of Δ_2. Assume that for any $y \in R$ either $\delta_1(y) = 0$ or $\delta_2(y) = 0$. Then either $\Delta_1 = 0$ or $\Delta_2 = 0$.*

Proof. We remark that if $\delta_1(y) = 0$ for all $y \in R$, then

$$0 = \delta_1(x + y) = \delta_1(x) + \delta_1(y) + 2\Delta_1(x, y) = 2\Delta_1(x, y), \quad \forall x, y \in R$$

that is $\Delta_1 = 0$. Analogously, if $\delta_2(y) = 0$ for all $y \in R$, then $\Delta_2 = 0$.

We now assume that both $\delta_1 \neq 0$ and $\delta_2 \neq 0$ and prove that a number of contradictions follows. In other words, we suppose there exists $x, y \in R$ such that $\delta_1(x) \neq 0$ and $\delta_2(y) \neq 0$, so that, by the hypothesis of the present Lemma, $\delta_2(x) = 0$ and $\delta_1(y) = 0$. We divide the argument into two cases:

Case 1. $\delta_1(x + y) = 0$.

In this case

$$\delta_1(x) + 2\Delta_1(x, y) = 0. \tag{30}$$

Moreover, if $\delta_1(x - y) = 0$, then

$$\delta_1(x) - 2\Delta_1(x, y) = 0 \tag{31}$$

and comparing (31) with (30), we have the contradiction $\delta_1(x) = 0$. Similarly, if $\delta_1(x - 2y) = 0$, then

$$\delta_1(x) - 4\Delta_1(x, y) = 0 \tag{32}$$

and comparing (32) with (30) we have again the contradiction $\delta_1(x) = 0$.

Thus both $\delta_1(x - y) \neq 0$ and $\delta_1(x - 2y) \neq 0$, that is $\delta_2(x - y) = 0$ and $\delta_2(x - 2y) = 0$, that is, respectively,

$$\delta_2(y) - 2\Delta_2(x, y) = 0 \tag{33}$$

and

$$4\delta_2(y) - 4\Delta_2(x, y) = 0. \tag{34}$$

Once again comparing (33) with (34), we get the contradiction $\delta_2(y) = 0$.

Case 2. $\delta_1(x + y) \neq 0$.

In this case

$$0 = \delta_2(x + y) = \delta_2(y) + 2\Delta_2(x, y) = 0. \tag{35}$$

Moreover, if $\delta_2(x - y) = 0$, then

$$\delta_2(y) - 2\Delta_2(x, y) = 0, \tag{36}$$

and comparing (36) with (35), we have $\delta_2(y) = 0$, which is a contradiction. Similarly, if $\delta_2(x - 2y) = 0$, then

$$4\delta_2(y) - 4\Delta_2(x, y) = 0, \tag{37}$$

and comparing (37) with (35) we have again $\delta_2(y) = 0$, a contradiction.

On the other hand, in case both $\delta_2(x - y) \neq 0$ and $\delta_2(x - 2y) \neq 0$, then $\delta_1(x-y) = 0$ and $\delta_1(x - 2y) = 0$, that is, respectively, $\delta_1(x) - 2\Delta_1(x, y) = 0$ and $\delta_1(x) - 4\Delta_1(x, y) = 0$, thus the contradiction $\delta_1(x) = 0$ follows. □

Proof of Theorem 3. By our assumption,

$$\delta_1(x)\delta_2(x) = 0, \quad \forall x \in R. \tag{38}$$

Firstly, we fix some element $x_0 \in R$ such that $\delta_1(x_0) \in Z(R)$. Then, by (38), either $\delta_1(x_0) = 0$ or $\delta_2(x_0) = 0$. Analogously, if $\delta_2(x_0) \in Z(R)$, then either $\delta_1(x_0) = 0$ or $\delta_2(x_0) = 0$.

Hence, if we suppose that for any $x \in R$ either $\delta_1(x) \in Z(R)$ or $\delta_2(x) \in Z(R)$, then we have that for any $x \in R$ either $\delta_1(x) = 0$ or $\delta_2(x) = 0$, and the conclusion follows from Lemma 6.

Now we assume that there exists $y_0 \in R$ such that $0 \neq \delta_1(y_0) \notin Z(R)$ and $0 \neq \delta_2(y_0) \notin Z(R)$. In (38) replace x by $y_0 + x$, then

$$\begin{aligned}\delta_1(x)\delta_2(y_0) + \delta_1(y_0)\delta_2(x) + 2\delta_1(x)\Delta_2(x, y_0) + 2\delta_1(y_0)\Delta_2(x, y_0) + 2\Delta_1(x, y_0)\delta_2(x)\\ + 2\Delta_1(x, y_0)\delta_2(y_0) + 4\Delta_1(x, y_0)\Delta_2(x, y_0) = 0,\ \forall x \in R. \end{aligned}\tag{39}$$

On the other hand, replacing x by $y_0 - x$ in (38), we also have

$$\begin{aligned}\delta_1(x)\delta_2(y_0) + \delta_1(y_0)\delta_2(x) - 2\delta_1(x)\Delta_2(x, y_0) - 2\delta_1(y_0)\Delta_2(x, y_0) - 2\Delta_1(x, y_0)\delta_2(x)\\ - 2\Delta_1(x, y_0)\delta_2(y_0) + 4\Delta_1(x, y_0)\Delta_2(x, y_0) = 0,\ \forall x \in R. \end{aligned}\tag{40}$$

By comparing (39) with (40), we get

$$\delta_1(x)\delta_2(y_0) + \delta_1(y_0)\delta_2(x) + 4\Delta_1(x, y_0)\Delta_2(x, y_0) = 0, \quad \forall x \in R. \tag{41}$$

Substituting x with $x + y_0$ in (41), using both (38) and (41), and since $\mathrm{char}(R) \neq 2, 3$, it follows that

$$\delta_1(y_0)\Delta_2(x, y_0) + \Delta_1(x, y_0)\delta_2(y_0) = 0, \quad \forall x \in R. \tag{42}$$

Here we introduce the following notations:

$$\begin{aligned} F_1(x) &= \Delta_1(x, y_0),\ \forall x \in R, \quad f_1(x) = D_1(x, y_0),\ \forall x \in R\\ F_2(x) &= \Delta_2(x, y_0),\ \forall x \in R, \quad f_2(x) = D_2(x, y_0),\ \forall x \in R\\ 0 \neq \delta_1(y_0) &= a \notin Z(R), \quad 0 \neq \delta_2(y_0) = b \notin Z(R). \end{aligned}\tag{43}$$

Notice that F_1, F_2 are generalized skew derivations of R with associated automorphism α and associated skew derivations f_1, f_2 respectively. Moreover, by (42) we have that

$$aF_2(x) + F_1(x)b = 0, \quad \forall x \in R. \tag{44}$$

Application of [4, Theorem 1 and Corollary 1] implies that there exists an invertible element $s \in Q$ such that $\alpha(x) = sxs^{-1}$, for all $x \in R$, and one of the following holds:

(1) $F_1(x) = [a, sxs^{-1}]s$, $F_2(x) = s[b, x]$ for any $x \in R$, with $s^{-1}asb \in C$;
(2) there exists $\eta \in C$ such that $F_1(x) = sx + \eta[a, sxs^{-1}]s$, $F_2(x) = sx + \eta s[b, x]$ for any $x \in R$, with $as + sb = 0$ and $\eta s^{-1}asb - b \in C$.

Case 1. $F_1(x) = [a, sxs^{-1}]s$, $F_2(x) = s[b, x]$ for any $x \in R$, with $s^{-1}asb \in C$. For any $x, t \in R$ we have that

$$F_1(xt) = [a, sxts^{-1}]s = asxt - sxts^{-1}as, \tag{45}$$

and also

$$F_1(xt) = F_1(x)t + sxs^{-1}f_1(t) = asxt - sxs^{-1}ast + sxs^{-1}f_1(t). \tag{46}$$

By (45) and (46) we get $s^{-1}f_1(t) = [s^{-1}as, t]$, that is

$$f_1(t) = s[s^{-1}as, t], \forall t \in R. \tag{47}$$

Moreover,

$$F_2(xt) = s[b, xt] = s[b, x]t + sx[b, t] \tag{48}$$

and also

$$F_2(xt) = F_2(x)t + sxs^{-1}f_2(t) = s[b, x]t + sxs^{-1}f_2(t). \tag{49}$$

By (48) and (49) we get $s^{-1}f_2(t) = [b, t]$, that is

$$f_2(t) = s[b, t], \forall t \in R. \tag{50}$$

Using (49) and (50), it follows that

$$af_2(x) + f_1(x)b = 0, \forall x \in R. \tag{51}$$

We recall that, by Fact 1, there exist $c_1, c_2 \in Q$ such that $F_1(x) = c_1x + f_1(x)$ and $F_2(x) = c_2x + f_2(x)$, for any $x \in R$. Then, by (44) and (51) one has $ac_2x + c_1xb = 0$, for all $x \in R$. Since $0 \neq b \notin C$, we have $c_1 = ac_2 = 0$. Denote d_1 the trace of D_1, hence

$$F_1(x) = f_1(x), \quad \Delta_1(x, y_0) = D_1(x, y_0), \quad d_1(x) = \delta_1(x), \quad \forall x \in R,$$

and

$$\delta_1(y_0)F_2(x) = \delta_1(y_0)f_2(x) = \delta_1(y_0)D_2(x, y_0), \quad \forall x \in R.$$

Thus (44) reduces to

$$aD_2(x, y_0) + D_1(x, y_0)b = 0, \forall x \in R. \tag{52}$$

Replacing in (52) x by xt and using again (52), we get

$$D_1(x, y_0)[b, t] + [a, \alpha(x)]D_2(t, y_0) = 0, \forall x, t \in R. \tag{53}$$

Now we substitute x with zx in (53) and use again (53), then

$$D_1(z, y_0)x[b, t] + [a, \alpha(z)]\alpha(x)D_2(t, y_0) = 0, \forall x, t, z \in R. \tag{54}$$

Since $\alpha(x) = sxs^{-1}$, by replacing x with xs in (54), we have

$$D_1(z, y_0)xs[b, t] + [a, szs^{-1}]sxD_2(t, y_0) = 0, \forall x, t, z \in R. \tag{55}$$

Here we remark that

$$D_2(t, y_0) = f_2(t) = s[b, t] \quad \text{and} \quad D_1(z, y_0) = F_1(z) = [a, szs^{-1}]s,$$

therefore, we may write (55):

$$2[a, szs^{-1}]sxs[b, t] = 0, \forall x, t, z \in R.$$

By the primeness of R it follows that: either $[a, szs^{-1}]s = 0$ for all $z \in R$, which implies $a \in C$; or $s[b, t] = 0$, for all $t \in R$, that is $b \in C$. In any case, we have a contradiction.

Case 2. There exists $\eta \in C$ such that $F_1(x) = sx + \eta[a, sxs^{-1}]s$, $F_2(x) = sx + \eta s[b, x]$ for any $x \in R$, with $as + sb = 0$ and $\eta s^{-1}asb - b \in C$. Note that $b \notin C$ implies $\eta \neq 0$. Moreover, it is easy to see that $\eta b^2 + b = \lambda \in C$ and also that $bs^{-1}b = 0$ (since $ab = 0$).

For any $x, t \in R$, we have that

$$F_1(xt) = sxt + \eta[a, sxts^{-1}]s = sxt + \eta asxt - \eta sxts^{-1}as \tag{56}$$

and also

$$F_1(xt) = F_1(x)t + sxs^{-1}f_1(t) = sxt + \eta[a, sxs^{-1}]st + sxs^{-1}f_1(t). \tag{57}$$

Comparing (56) and (57), we get

$$sx\left(-\eta ts^{-1}as + \eta s^{-1}ast - s^{-1}f_1(t)\right) = 0$$

and by the primeness of R, and since $as = -sb$, it follows

$$f_1(t) = \eta s[t, b], \quad \forall t \in R. \tag{58}$$

Moreover,

$$F_2(xt) = sxt + \eta s[b, xt] = sxt + \eta s[b, x]t + \eta sx[b, t] \tag{59}$$

and also

$$F_2(xt) = F_2(x)t + sxs^{-1}f_2(t) = (qx + \eta s[b, x])t + sxs^{-1}f_2(t). \tag{60}$$

By (59) and (60) we get $sx(\eta[b, t] - s^{-1}f_2(t))$, and, by the primeness of R,

$$f_2(t) = \eta s[b, t], \quad \forall t \in R. \tag{61}$$

We note that $f_1 = -f_2$. As above, there exists $c_1 \in Q$ such that $F_1(x) = c_1x + f_1(x) = c_1x + \eta s[x, b]$, for all $x \in R$. Thus we may write $c_1x + \eta s[x, b] = F_1(x) = sx + \eta[a, sxs^{-1}]s$, and by computations it follows $c_1 = s$.

In other words, we obtain that $F_1(x) = sx + f_1(x) = sx + \eta s[x, b]$ and $F_2(x) = sx - f_1(x) = sx - \eta s[x, b]$, for any $x \in R$. According to (43), this means that $D_2(x, y_0) = -D_1(x, y_0)$, for any $x \in R$. By (42)

$$a\Delta_2(xz, y_0) + \Delta_1(xz, y_0)b = 0, \quad \forall x, z \in R, \tag{62}$$

so that, since $D_2 = -D_1$,

$$a\Delta_2(x, y_0)z - asxs^{-1}D_1(z, y_0) + \Delta_1(x, y_0)zb + sxs^{-1}D_1(z, y_0)b = 0, \forall x, z \in R. \tag{63}$$

Using (42) in (63), we have

$$-asxs^{-1}D_1(z, y_0) + \Delta_1(x, y_0)[z, b] + sxs^{-1}D_1(z, y_0)b = 0, \quad \forall x, z \in R \tag{64}$$

and right multiplying by $s^{-1}b$, left multiplying by bs^{-2}, and since $as = -sb$ and $bs^{-1}b = 0$, it follows that

$$bs^{-2}\Delta_1(x, y_0)bzs^{-1}b = 0, \quad \forall x, z \in R. \tag{65}$$

By the primeness of R, either $s^{-1}b = 0$, that is $b = 0$, which is a contradiction, or $bs^{-2}\Delta_1(x, y_0)b = 0$, for any $x \in R$. In this last case

$$0 = bs^{-2}\Big(sx + \eta s[x, b]\Big)bzs^{-1}b = \eta bs^{-1}xb^2, \quad \forall x \in R, \tag{66}$$

which implies $b^2 = 0$. Thus, right multiplying (42) by b, one has

$$a\Delta_2(x, y_0)b = 0, \quad \forall x \in R, \tag{67}$$

that is

$$0 = a\Big(sx - \eta s[x, b]\Big)b = -sbxb, \quad \forall x \in R, \tag{68}$$

which implies again the contradiction $b = 0$.

As consequence of Theorem 3, we also have the following:

Corollary 1. *Let R be a prime ring of characteristic different from two and three, $\Delta_1, \Delta_2 : R \times R \longrightarrow R$ two symmetric generalized skew biderivations. If $\Delta_1(x, y)\Delta_2(x, y) = 0$ for any $x, y \in R$, then either $\Delta_1 = 0$ or $\Delta_2 = 0$.*

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