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Title: Computing relative power integral bases in a family of quartic extensions of imaginary quadratic fields

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Let $M = \mathbb{Q}(\sqrt{-D})$ be an imaginary quadratic field with the ring of integers $\mathbb{Z}_M$, and let ξ be a root of the polynomial $f(x) = x^4 - 2cx^3 + 2x^2 + 2cx + 1$, where $c \in \mathbb{Z}_M \setminus \{0, \pm 2\}$ and $c \neq \pm 1$ if $D = 1$ or 3. We consider an infinite family of octic fields $K_c = M(\xi)$ with the ring of integers $\mathbb{Z}_{K_c}$. Our goal is to determine all generators of a relative power integral basis of $\mathcal{O} = \mathbb{Z}_M[\xi]$ over $\mathbb{Z}_M$. We show that our problem reduces to solving the system of relative Pellian equations $cV^2 - (c+2)U^2 = -2\mu$, $cZ^2 - (c-2)U^2 = 2\mu$, where μ is a unit in $\mathbb{Z}_M$. We solve the system completely and find that all non-equivalent generators of power integral bases of $\mathcal{O}$ over $\mathbb{Z}_M$ are given by $\alpha = \xi$, $2\xi - 2c\xi^2 + \xi^3$ for $|c| \geq 159108$ and $|c| \leq 1000$, $c \notin S_c$ (where S_c is a set of exceptional cases, $|S_c| = 28$). Also, we find that, in all the above cases, $\mathcal{O}$ admits no absolute power integral basis if $-D \equiv 2, 3 \pmod 4$.

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