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Title: Characterization of the Euler gamma function with the aid of an arbitrary mean

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We prove that a continuous function $f:(0,\infty)\to(0,\infty)$ satisfying the functional equation

$$f(x+1)=xf(x), \quad x>0, \quad f(1)=1,$$

is the Euler gamma function iff for some $a>0$ and a strict and continuous mean $M:(a,\infty)^2\to(a,\infty)$, the following inequality holds:

$$f(M(x,y))\,f\left(\frac{xy}{M(x,y)}\right)\le f(x)\,f(y), \quad x,y\in(a,\infty).$$

Taking for M the geometric mean $G(x,y)=\sqrt{xy}$, we obtain the result of [2] generalizing the classical Bohr–Mollerup theorem [1]. For $M=A$, where $A(x,y)=\frac{x+y}{2}$ is the arithmetic mean, the assumed inequality reduces to $f(A(x,y))\,f(H(x,y))\le f(x)\,f(y)$ for all $x,y>a$, where H is the harmonic mean, and the result gives a new characterization of the gamma function, involving the arithmetic and harmonic means.

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