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Erratum to the paper: "On the Diophantine equations $(x-1)^3 + x^5 + (x+1)^3 = y^n$ and $(x-1)^5 + x^3 + (x+1)^5 = y^n$"

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In the proof of [2, Theorem 1.2], we miss the case $5|x$ for n an odd prime. When $5 \nmid x$, since $\gcd(2x^2+1, x) = 1$, $\gcd(x^2+10, x) = 1$ or 2, one has $x = 2^\beta u^n$, $19^\alpha z^n = 2x^2 + 1 = 2^{2\beta+1} u^{2n} + 1$, and then Lemma 2.3 can be used to get the result. But when $5|x$, since $\gcd(x^2+10, x) = 5$ or 10, one has $x = 2^\beta \times 5^{n-1} u^n$, $19^\alpha z^n = 2x^2 + 1 = 2^{2\beta+1} \times 5^{2n-2} u^{2n} + 1$, and then the following lemma can be used to obtain the result.

Lemma 1. *The binomial Thue equation*

$$19^\alpha z^p - 2^{2\beta+1} \times 5^{2p-2} y^{2p} = 1$$

has no integer solutions with p an odd prime and $y \neq 0$, where $\alpha = 0, 1$ or $p-1$ and $\beta = 0$ or $p-1$.

Proof. When $\alpha = 0$, the lemma can be obtained from [1, Theorem 1.1].

When $\alpha = 1$ or $p-1$, and $p \geq 7$, we can associate the Frey curves

$$E : Y^2 = X(X+1)(X - 2^{2\beta+1} \times 5^{2p-2} y^{2p}).$$

There is a newform of level $N(E)_p = 190$ for $\beta = p-1$, or a newform of level $N(E)_p = 3040$ for $\beta = 0$ such that $E \sim_p f$. Then we can get $p \leq 5$. The case $p = 3, 5$ can be obtained by solving Thue equations using Magma. $\square$

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