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**Title:** Composantes isotypiques de pro- $p$ -extensions de corps de nombres et  $p$ -rationalité

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Let  $p$  be a prime number, and let  $K/k$  be a finite Galois extension of number fields with Galois group  $\Delta$  of order coprime to  $p$ . Let  $S$  be a finite set of non-Archimedean places of  $k$  including the set  $S_p$  of  $p$ -adic places, and let  $K_S$  be the maximal pro- $p$  extension of  $K$  unramified outside  $S$ . Let  $G := G_S/H$  be a quotient of  $G_S := \text{Gal}(K_S/K)$  on which  $\Delta$  acts trivially. Put  $X := H/[H, H]$ . In this paper, we study the  $\varphi$ -component  $X^\varphi$  of  $X$  for all  $\mathbb{Q}_p$ -irreducible characters  $\varphi$  of  $\Delta$ , and, in particular, by assuming the Leopoldt conjecture, we show that for all non-trivial characters  $\varphi$ , the  $\mathbb{Z}_p[[G]]$ -module  $X^\varphi$  is free if and only if the  $\varphi$ -component of the  $\mathbb{Z}_p$ -torsion of  $G_S/[G_S, G_S]$  is trivial. We also make a numerical study of the freeness of  $X^\varphi$  in cyclic extensions  $K/Q$  of degree 3 and 4 (by using families of polynomials given by Balady, Lecacheux, and more recently by Balady and Washington), but also in degree 6 dihedral extension over  $\mathbb{Q}$ : the results we get support a recent conjecture of Gras.

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