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Hereafter, R denotes a noncommutative division ring with centre Z, and $f\colon R \to R$ is a semi-linear additive map of R (in the sense given by N. Jacobson, or a more general condition given in the Introduction). In this article, we show that if f is power commuting, that is, (i) there is a positive integer m such that $[f(x), x^m] = 0$, all $x \in R$, then f is, in fact, commuting, that is, $[f(x), x] = 0$, all $x \in R$. More generally, suppose that (ii) for a fixed pair of positive integers m and n, $[f(x), x^m]_n = 0$, all $x \in R$. Again, we will show that f is commuting. Now, a doubly more liberal version of the latter condition is Condition (C), which asserts that for each x in R, $[f(x), x^{m(x)}]_{n(x)} = 0$, where $m(x)$ and $n(x)$ are both positive integers depending on x. Unless we are ready to condition appropriately the carrier R, the status of Condition (C) remains totally unknown. Granted R is algebraic over Z, in particular if R is finite dimensional over Z, we show here that if f is an endomorphism or anti-endomorphism of R, then from Condition (C) follows again that f is commuting.

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