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**Title:** An asymptotic formula concerning Lehmer numbers

**Author(s):** James P. Jones and Péter Kiss

Let  $L_n$ ,  $n = 0, 1, 2, \dots$ , be a Lehmer sequence defined by  $L_n = (\alpha^n - \beta^n)/(\alpha - \beta)$  for  $n$  odd and  $L_n = (\alpha^n - \beta^n)/(\alpha^2 - \beta^2)$  for  $n$  even, where  $(\alpha + \beta)^2 = A$  and  $\alpha\beta = -B$  are fixed rational integers and  $|\alpha| \geq |\beta|$ . Let  $m$  be an integer  $> 1$  and define the sequence  $(M_n)$  of integers by  $M_n = L_{mn}/L_n$  for  $n > 0$ . We prove that

$$\frac{\log |M_1 \cdot M_2 \cdots M_N|}{\log [M_1, M_2, \dots, M_N]} = \frac{m-1}{6(1-w)(m - \prod_{p|m} \frac{p}{p+1})} \pi^2 + O\left(\frac{\log N}{N}\right)$$

for sufficiently large  $N$ , where  $w = \log((A, B))/2 \cdot \log |\alpha|$  and  $[M_1, M_2, \dots]$  denotes the least common multiple of  $M_1, M_2, \dots$ . This result is a generalization and an improvement of a formula given by J. P. Bézivin.

**Address:**

James P. Jones  
Department of Mathematics and Statistics  
The University of Calgary  
2500 University Drive N.W.  
Calgary, Alberta, T2N 1N4  
Canada

**Address:**

Péter Kiss  
Department of Mathematics  
Eszterházy Károly University  
Leányka u. 4  
3301 Eger  
Hungary