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Title: Interiors of continuous images of self-similar sets with overlaps

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Let K be the attractor of the following iterated function system

$$\{S_1(x) = \lambda x, S_2(x) = \lambda x + c - \lambda, S_3(x) = \lambda x + 1 - \lambda\},$$

where $S_1(I) \cap S_2(I) \neq \emptyset, (S_1(I) \cup S_2(I)) \cap S_3(I) = \emptyset$, and $I = [0, 1]$ is the convex hull of K. Let $d_1 = \dfrac{1-c-\lambda}{\lambda} < \dfrac{1}{1-c-\lambda} = d_2$. Suppose that f is a continuous function defined on an open set $U \subset \mathbb{R}^2$. Denote the image

$$f_U(K, K) = \{f(x, y) : (x, y) \in (K \times K) \cap U\}.$$

If $\partial_x f$, $\partial_y f$ are continuous on U, and there is a point $(x_0, y_0) \in (K \times K) \cap U$ such that

$$\left|\frac{\partial_y f|_{(x_0,y_0)}}{\partial_x f|_{(x_0,y_0)}}\right| \in (d_1, d_2) \quad \text{or} \quad \left|\frac{\partial_x f|_{(x_0,y_0)}}{\partial_y f|_{(x_0,y_0)}}\right| \in (d_1, d_2),$$

then $f_U(K, K)$ contains an interval. As a result, we let $c = \lambda = \dfrac{1}{3}$, and if

$$f(x, y) = x^\alpha y^\beta (\alpha\beta \neq 0), \quad x^\alpha \pm y^\alpha (\alpha \neq 0), \quad \sin(x)\cos(y), \quad \text{or } x\sin(xy),$$

then $f_U(C, C)$ contains an interval, where C is the middle-third Cantor set.

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