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Sincov's inequalities on topological spaces

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Abstract. Assume that X is a non-empty set, and T and S are real or complex mappings defined on the product $X \times X$. Additive and multiplicative Sincov's equations are:

$$T(x,z) = T(x,y) + T(y,z), \qquad x,y,z \in X$$

and

$$S(x,z) = S(x,y) \cdot S(y,z), \qquad x,y,z \in X,$$

respectively. In the present paper, we study three related inequalities. We begin with functional inequality

$$G(x,z) \leq G(x,y) \cdot G(y,z), \qquad x,y,z \in X,$$

and assume that X is a topological space and $G\colon X \times X \to \mathbb{R}$ is a continuous mapping. In some our statements a considerably weaker regularity than continuity of G is needed. Next, we study the reverse inequality:

$$F(x,z) \geq F(x,y) \cdot F(y,z), \qquad x,y,z \in X,$$

as well as the additive inequality:

$$H(x,z) \leq H(x,y) + H(y,z), \qquad x,y,z \in X.$$

A corollary for generalized metric is derived.

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