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Title: Strong arithmetic property of certain Stern polynomials

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Let $B_n(t)$ be the n-th Stern polynomial, i.e., the n-th term of the sequence defined recursively as $B_0(t) = 0, B_1(t) = 1$ and $B_{2n}(t) = tB_n(t), B_{2n+1}(t) = B_n(t) + B_{n-1}(t)$ for $n \in \mathbb{N}$. It is well known that the i-th coefficient in the polynomial $B_n(t)$ counts the number of hyperbinary representations of $n-1$ containing exactly i digits 1. In this note we investigate the existence of odd solutions of the congruence

$$B_n(t) \equiv 1 + rt\frac{t^{e(n)} - 1}{t - 1} \pmod{m},$$

where $m \in \mathbb{N}_{\geq 2}$ and $r \in \{0, \dots, m-1\}$ are fixed and $e(n) = \mathrm{degB_n(t)}$. We prove that for $m = 2$ and $r \in \{0, 1\}$ and for $m = 3$ and $r = 0$, there are infinitely many odd numbers n satisfying the above congruence. We also present results of some numerical computations.

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