Year: 2020 **Vol.: 96** **Fasc.: 3-4**

Title: On k-antichains in the unit n-cube

Author(s): Christos Pelekis and Václav Vlasák

A *chain* in the unit n-cube is a set $C \subset [0,1]^n$ such that for every $\mathbf{x} = (x_1, \dots, x_n)$ and $\mathbf{y} = (y_1, \dots, y_n)$ in C, we either have $x_i \leq y_i$ for all $i \in [n]$, or $x_i \geq y_i$ for all $i \in [n]$. We consider subsets A, of the unit n-cube $[0,1]^n$, that satisfy

$$\operatorname{card}(A \cap C) \leq k, \quad \text{for all chains } C \subset [0,1]^n,$$

where k is a fixed positive integer. We refer to such a set A as a k-antichain. We show that the $(n-1)$-dimensional Hausdorff measure of a k-antichain in $[0,1]^n$ is at most kn and that the bound is asymptotically sharp. Moreover, we conjecture that there exist k-antichains in $[0,1]^n$ whose $(n-1)$-dimensional Hausdorff measure equals kn, and we verify the validity of this conjecture when $n = 2$.

Address:
Christos Pelekis
Institute of Mathematics
Czech Academy of Sciences
Žitná 25
115 67 Praha 1
Czech Republic

Address:
Václav Vlasák
Faculty of Mathematics and Physics
Charles University
Sokolovská 83
18675 Praha 8
Czech Republic