

**Title:** A sharp trigonometric double inequality

**Author(s):** Yongbeom Kim, Tuo Yeong Lee, Vengat S., Hui Xiang Sim and Jay Kin Heng Tai

We prove that

$$\left(\frac{5-\sqrt{5}}{8}\right)^{\frac{3}{2}} + \frac{1}{2} \sin^3 \frac{8\pi}{5} \leq \sum_{k=1}^n \frac{\sin^3 k\theta}{k} \leq 1 \quad \text{for all integers } n \geq 1 \text{ and } \theta \in (0, \pi),$$

where both bounds are sharp. This gives an affirmative answer to a conjecture of Alzer and Koumandos.

**Address:**

Yongbeom Kim  
Department of Mathematics and Statistics  
NUS High School of Math and Science  
20 Clementi Avenue 1  
Singapore 129957  
Republic of Singapore

**Address:**

Tuo Yeong Lee  
NUS High School of Math and Science

**Address:**

Vengat S.  
NUS High School of Math and Science

**Address:**

Hui Xiang Sim  
NUS High School of Math and Science

**Address:**

Jay Kin Heng Tai  
NUS High School of Math and Science