Title: On k-generalized Fibonacci numbers with negative indices

Author(s): Attila Pethő

In these notes we study k-generalized Fibonacci sequences, $(F_n^{(k)})_{n\in\mathbb{Z}}$, with positive and negative indices. Denote by $T_k(x)$ its characteristic polynomial. Our most interesting finding is that if k is even, then the absolute value of the second real root of $T_k(x)$ is minimal among the roots. Combining this with a deep result of Bugeaud and Kaneko [6], we prove that there are only finitely many perfect powers in $(F_n^{(k)})_{n\in\mathbb{Z}}$, provided k is even. Another consequence is that if k and l denote even integers, then the equation $F_m^{(k)} = \pm F_n^{(l)}$ has only finitely many effectively computable solutions in $(n, m) \in \mathbb{Z}^2$. In the case $k = l = 4$, we establish all solutions of this equation.

Address:
Attila Pethő
Department of Computer Science
University of Debrecen
P. O. Box 400
H-4002 Debrecen
Hungary