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Title: Some functional lower bounds for Fejér's sine polynomial

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Let $\alpha \in \mathbb{R}$. We prove that

$$\sum_{k=1}^{n} \frac{\sin k\theta}{k} \geqslant \left(\frac{335}{576} + \frac{31}{128}\alpha\right)\sin\theta + \left(\frac{39}{128} + \frac{5}{32}\alpha\right)\sin 2\theta + \left(\frac{113}{2304} + \frac{21}{256}\alpha\right)\sin 3\theta + \left(\frac{25}{256} - \frac{5}{64}\alpha\right)\sin 4\theta + \left(\frac{125}{2304} - \frac{25}{256}\alpha\right)\sin 5\theta > \theta^2\left(\cot\frac{\theta}{2} - \frac{\pi-\theta}{2}\right)$$

for every integer $n \geqslant 2$ and $\theta \in (0, \pi)$ if and only if

$$\frac{\pi^2}{24} \leqslant \alpha \leqslant \alpha_0,$$

where α_0 denotes the unique real zero of the cubic polynomial

$$41721661440t^3 - 57761574336t^2 + 46230194016t - 13479325596.$$

Both bounds for α are sharp. This sharpens a result due to Alzer and Koumandos.

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