On simple groups.

By J. SZÉP in Szeged.

The following theorem holds:

Theorem 1. [1]) *A simple group of finite order can have a subgroup of index $i(>1)$ only in the case, if the order of the group is a divisor of $i!$.*

Theorem 2. *The alternating group of order $\frac{n!}{2}$ $(n \geqq 5)$ can have a subgroup of prime index only in the case if n is a prime number.*

Proof. If the alternating group $\mathfrak{A}$ has a subgroup of index p then it has the order $\frac{n!}{2p}$. As easily seen, if n is not a prime number, $(p-1)!$ is not divisible by $\frac{n!}{2p}$ and thus theorem 1 would imply that $\mathfrak{A}$ is not a simple group.

Theorem 3. [1]) *If a simple group $\mathfrak{G}$ has a subgroup of index p, where p is a prime number, it follows that the prime number p is the maximal prime factor of the order of $\mathfrak{G}$ and the order of $\mathfrak{G}$ contains it only on the first power.*

It is known, that the alternating group of order $\frac{n!}{2}$ has a subgroup of the index n. From this remark and from theorem 3 follows the

Corollary. *If a simple group $\mathfrak{G}$ has a subgroup $\mathfrak{H}$ with a prime index p, $\mathfrak{G}$ can be represented as a product of the subgroups $\mathfrak{H}$ and $\mathfrak{P}$ where $\mathfrak{P}$ denotes the Sylow group of order p.*

On research of construction of this group we have a method [2].)

(Received October 15, 1949.)

---

[1]) This theorem is a direct consequence of theorem 100 in A. SPEISER: *Theorie der Gruppen von endlicher Ordnung,* 3. ed. (Berlin, 1937).

[2]) L. RÉDEI: Die Anwendung des schiefen Produktes in der Gruppentheorie, *J. reine angew. Math.* (Under press.)

J. SZÉP: On the structure of groups which can be represented as the product of two subgroups. *Acta Sci. Math. (Szeged).* (Under press.)

G. ZAPPA: Sulla costruzione dei gruppi prodotto di due dati sottogruppi permutabili tra loro. *Atti del secondo Congresso dell' Unione Matematica Italiana,* Bologna **4.** (1940), pp. 119—125.