On the inequality of Mathieu.

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The inequality

$$(*) \qquad \sum_{n=1}^{\infty} \frac{2n}{(n^2+x^2)^2} < \frac{1}{x^2},$$

conjectured by E. MATHIEU ([4], Chap. X.), was proved in a weaker form by K. SCHRÖDER ([5], pp. 258—260) and later in its original form by L. BERG [1], O. EMERSLEBEN [3] and by J. G. VAN DER CORPUT and L. O. HEFLINGER [2]. Incidentally the latter authors show, that O. EMERSLEBEN's proof contains a fault which however can be corrected.

Each of these proofs uses more or less intricate analytic tools.

The purpose of this note is to point out that MATHIEU's inequality can be obtained from the elementary relations

$$\frac{1}{\left(n-\frac{1}{2}\right)^2+x^2-\frac{1}{4}} - \frac{1}{\left(n+\frac{1}{2}\right)^2+x^2-\frac{1}{4}} = \frac{2n}{(n^2+x^2-n)(n^2+x^2+n)} >$$

$$> \frac{2n}{(n^2+x^2)^2} \qquad (x \neq 0,\ n=1,2,\ldots)$$

simply by summing from $n=1$ to $n=\infty$.

As a counterpart to (*) we can obtain in a similar way a lower estimate for the left side of (*). From

$$\frac{1}{\left(n-\frac{1}{2}\right)^2+x^2+\frac{1}{4}} - \frac{1}{\left(n+\frac{1}{2}\right)^2+x^2+\frac{1}{4}} = \frac{2n}{(n^2+x^2)^2+x^2+\frac{1}{4}} <$$

$$< \frac{2n}{(n^2+x^2)^2} \qquad (n=1,2,\ldots)$$

it follows namely

$$\frac{1}{x^2+\frac{1}{2}} < \sum_{n=1}^{\infty} \frac{2n}{(n^2+x^2)^2}.$$

Bibliography.

[1] L. Berg, Über eine Abschätzung von Mathieu, *Math. Nachr.* **7** (1952), 257—259.

[2] J. G. van der Corput and L. O. Heflinger, On the inequality of Mathieu, *Indag. Math.* **18** (1956), 15—20.

[3] O. Emersleben, Über die Reihe $\Sigma k(k^2+c^2)^{-2}$, *Math. Ann.* **125** (1952), 165—171.

[4] E. Mathieu, Traité de physique mathématique, t. VI—VII. Seconde partie, *Paris*, 1890.

[5] K. Schröder, Das Problem der eingespannten rechteckigen elastischen Platte, *Math. Ann.* **121** (1949), 247—326.

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