Correction to my paper "The quasi-series decomposition of two-terminal graphs"

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1. Dr. G. POLLÁK has kindly called my attention that the discussion of Case $4/b/\delta$ of the proof of Theorem 1 in my cited paper is incomplete. In order to correct the proof, I shall now point out a new lemma and give a more detailed treatment of the final part of the mentioned proof.

2. Beside the lemma exposed in § 4 of [1], we need also

Lemma 11. *Let $\mathfrak{G}$ be an indecomposable graph having at least three final edges. Let k_1 and $k_2(BQ)$ be two final edges of $\mathfrak{G}$; assume that B is an inner vertex of each 2-subgraph of $\mathfrak{G}$. Then there exists a chain $a(PB)$ in $\mathfrak{G}$ such that a contains k_1 but it does not contain k_2.*

PROOF. If we delete k_2 in $\mathfrak{G}$ (for a moment), then the resulting 2-graph is likewise indecomposable. Hence $\mathfrak{G}$ has two disjoint paths b, c which do not contain k_2. Furthermore, there exists a (possibly degenerated) chain $d(CB)$ such that

d contains no terminal of $\mathfrak{G}$,

C is an inner vertex in b or in c, and

d has no vertex, different from C, which occurs in b or in c.

The symmetry makes possible to suppose that C is contained in c.

Case 1: k_1 is contained in b or c. Then the chain

$$b \cdot c^{-1}[QC] \cdot d \quad .$$

satisfies the conclusion of the lemma.

Case 2: neither b nor c contains k_1. Let e be a path containing k_1. Denote by E the last vertex of e which differs from Q and is contained in one of b, c, d. We can distinguish five possibilities according to the situation of E:

$E=P$,

E is an inner vertex of b,

E is an inner vertex of $c[PC]$,

E is a vertex of $c[CQ]$,

E is a vertex of d.

According to the five possibilities enumerated, one of the chains

$$e\cdot c^{-1}[QC]\cdot d,$$
$$b[PE]\cdot e[EQ]\cdot c^{-1}[QC]\cdot d,$$
$$c[PE]\cdot e[EQ]\cdot c^{-1}[QC]\cdot d,$$
$$b\cdot e^{-1}[QE]\cdot c^{-1}[EC]\cdot d,$$
$$b\cdot e^{-1}[QE]\cdot d[EB]$$

fulfils the conclusion, respectively.

3. On p. 104 of [1], the final section of the proof of Case $4/b/\delta$ (beginning with the words "There are two alternatives") should be replaced by what follows:

Denote by $\mathfrak{H}_1$ and $\mathfrak{H}_2$ the narrowest (non-trivial) 2-subgraphs of $\mathfrak{G}^*$ which contain k_1 and k_2, respectively. It suffices to study only the case $\mathfrak{H}_1 \supseteqq \mathfrak{H}_2$. Denote by $\mathfrak{H}^*$ the widest proper 2-subgraph of $\mathfrak{H}_1$ and by $\mathfrak{K}$ the narrowest 2-subgraph satisfying $\mathfrak{H}_1 \supseteqq \mathfrak{K} \supseteqq \mathfrak{H}_2$ (provided that such a $\mathfrak{H}^*$ or $\mathfrak{K}$ does exist). The graphs $\mathfrak{H}_1/\mathfrak{H}^*$ and $\mathfrak{K}/\mathfrak{H}_2$ are irreducible or they consist of two edges. The subsequent seven situations are possible:

(i) $\mathfrak{H}_1 = \mathfrak{H}_2$,

(ii) $\mathfrak{H}_1/\mathfrak{H}^*$ (exists and) consists of two parallel-composed edges, moreover $\mathfrak{H}_2 \subseteqq \mathfrak{H}^*$,

(iii) $\mathfrak{H}_1/\mathfrak{H}^*$ is irreducible, $\mathfrak{H}_2 = \mathfrak{H}^*$, k_2 is a series component of $\mathfrak{H}_2$,

(iv) $\mathfrak{H}_1/\mathfrak{H}^*$ is irreducible, $\mathfrak{H}_2 = \mathfrak{H}^*$, $\mathfrak{H}_2$ is indecomposable,

(v) $\mathfrak{H}_1/\mathfrak{H}^*$ is irreducible, $\mathfrak{H}_2 \subset \mathfrak{H}^*$, and $\mathfrak{K}/\mathfrak{H}_2$ consists of two parallel-composed edges,

(vi) $\mathfrak{H}_1/\mathfrak{H}^*$ is irreducible, $\mathfrak{H}_2 \subset \mathfrak{H}^*$, and $\mathfrak{K}/\mathfrak{H}_2$ consists of two series-composed edges,

(vii) $\mathfrak{H}_1/\mathfrak{H}^*$ and $\mathfrak{K}/\mathfrak{H}_2$ are irreducible, $\mathfrak{H}_2 \subset \mathfrak{H}^*$.

In each of these cases we are going to point out that there exists a chain in $\mathfrak{G}^*$ between P and A such that both k_1 and k_2 occur in it.

If (i) is valid, then k_1 and k_2 form a separating pair in $\mathfrak{H}_1$; we conclude by Lemma 4. If (ii) holds, then there exists a chain in $\mathfrak{H}_1$ between Q_2 and A such that this chain contains k_1 and k_2 (Lemma 10); it can be completed by a suitable path of $\mathfrak{G}^*$ to a chain connecting P and A. (We have utilized Lemma 5, too; this result must be kept in mind also in what follows.) In cases (iii)—(vii) k_1 and h^* are final edges in $\mathfrak{H}_1/\mathfrak{H}^*$. If (iii) is true, then it suffices to consider the possibility when k_1, k_2 do not form a separating pair in $\mathfrak{H}_1$; $\mathfrak{H}_1/\mathfrak{H}_2$ has at least three final edges, thus we can apply Lemma 11 in $\mathfrak{H}_1/\mathfrak{H}_2$ for k_1 and h_2. Among the remaining four possibilities, if (v) is valid, then Lemma 8 is applicable in $\mathfrak{H}_2$; if one of (iv), (vi), (vii) holds, we can utilize Lemma 10.

Reference

[1] A. Ádám, The quasi-series decomposition of two-terminal graphs, Publ. Math. Debrecen, **10** (1963), 96—107.

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