# Remarks on a paper of C. J. Mozzochi

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*To Professor András Rapcsák on his 60th birthday*

In what follows we are going to make some remarks concerning C. J. MOZZOCHI's paper [2]. We shall freely use the terminology and notations of that paper, as well as those of the monograph [1].

## I

After having defined symmetric generalized proximity spaces $(X, \delta)$, MOZZOCHI proceeds to show that a topology for $X$ can be obtained from $\delta$ in a natural way, by considering a point $x$ as belonging to the closure of a set $A$ iff $x\delta A$. (See [2], Theorem (1.9).)

By definition a symmetric generalized proximity on $X$ is a relation on $\mathbf{P}(X)$ satisfying the five conditions (P.1)—(P.5). This set of conditions is more than what is needed if we are concerned only with deriving from $\delta$ a topology for $X$. As a matter of fact, if we retain only conditions (P.2) and (P.3), we still are able to obtain a result similar to Mozzochi's Theorem (1.9):

Let $\delta$ be a relation on $P(X)$ satisfying

(P.2) $C\delta(A\cup B)$ if and only if $C\delta A \vee C\delta B$;

(P.3) $A\bar{\delta}\emptyset$ for any $A\subseteq X$.

Put

$$A\in\mathscr{F} \quad \text{if and only if} \quad x\delta A \quad \text{implies} \quad x\in A.$$

We are now able to prove the following

Proposition 1. *The class $\mathscr{F}$ is the class of closed sets of a topology on $X$. In this topology*

$$A_\delta = \{x \mid x\delta A\} \subseteq \bar{A} \quad \text{for any} \quad A \subseteq X.$$

PROOF. $\emptyset\in\mathscr{F}$ since $x\delta\emptyset$ cannot occur; $X\in\mathscr{F}$ since always $x\in X$.

$$A, B\in\mathscr{F} \quad \text{implies} \quad A\cup B\in\mathscr{F}:$$

Indeed, if $x\delta A\cup B$, then $x\delta A$ and/or $x\delta B$ by (P.2). If, say, $x\delta A$, then $x\in A$ hence $x\in A\cup B$.

For any $\Gamma \neq \emptyset$, $A_\gamma \in \mathscr{F} (\gamma \in \Gamma)$ implies $\cap \{A_\gamma | \gamma \in \Gamma\} \in \mathscr{F}$:

Indeed, again by (P.2), $x\delta \cap \{A_\gamma | \gamma \in \Gamma\}$ implies $x\delta A_\gamma \cup \cap \{A_\gamma | \gamma \in \Gamma\}$, i.e. $x\delta A_\gamma$ for any (fixed) $\gamma \in \Gamma$. Hence $x \in A_\gamma$ $(\gamma \in \Gamma)$ and so

$$x \in \cap \{A_\gamma | \gamma \in \Gamma\}.$$

We still have to show that $A_\delta \subseteqq \bar{A}$, where

$$A_\delta = \{x | x\delta A\} \quad \text{and} \quad \bar{A} = \cap \{F | A \subseteqq F \in \mathscr{F}\}.$$

If $A \subseteqq F \in \mathscr{F}$, then $x\delta A \Rightarrow x\delta F \Rightarrow x \in F$, and this being true for any closed superset of $A$, $x\delta A$ implies $x \in \bar{A}$. ∎

*Remark*. By weakening a set of conditions, we usually diminish its ability to rule out trivial or degenerate cases. The situation arising if we retain from among (P.1)—(P.5) only the two conditions (P.2) and (P.3), is no exception to this rule. Consider e.g. the following "distantness" (rather than "proximity") function:

For subsets of an unbounded metric space, put

$$A = \emptyset \quad \text{and/or} \quad B = \emptyset \quad \text{implies} \quad A\bar{\delta}B,$$

and in case $A \neq \emptyset$, $B \neq \emptyset$, let

$$A\delta B \quad \text{iff} \quad \sup \{\varrho(x, y) | x \in A, y \in B\} > 1.$$

The relation $\delta$ so defined is neither a proximity nor a symmetric generalized proximity (no point is near to itself!), but it satisfies (P.2) and (P.3).

There is a slight difference between our Proposition 1. and Theorem (1.9) in [2], insofar as we have introduced a topology by exhibiting the class of closed sets, whereas (1.9) proceeds via the Kuratowski closure operator. If, in Proposition 1. we had $A_\delta = \bar{A}$ for any $A \subseteqq X$, this would show the two definitions to be equivalent not only under the conditions of Theorem (1.9) (which is clear!) but also in the more general situation considered in Proposition 1. However, we have been able to prove only $A_\delta \subseteqq \bar{A}$, and so we are led to ask, what conditions (if any) must be imposed on $\delta$ besides those underlying Proposition 1, in order to make valid $A_\delta = \bar{A}$ for any subset $A$ of $X$. The answer to this question is given by the following

Proposition 2. *Under the hypotheses of the foregoing proposition, the condition*

$$A_\delta = \bar{A} \quad \textit{for any} \quad A \subseteqq X$$

*will hold if and only if $\delta$ satisfies the following conditions:*

(P.4) *$x\delta x$ for any $x \in X$.*

(P.5a) *If $a\delta B$ and $b\delta C$ for all $b \in B$, then $a\delta C$.*

Proof. Suppose $A_\delta = \bar{A}$ true for any $A \subseteqq X$. Then $x \in \bar{x}$ implies $x\delta x$, i.e. (P.4) holds.

In order to establish (P.5a) i.e. the implication

$$a\delta B \,\&\, b\delta C (b \in B) \Rightarrow a\delta C,$$

we first remark that by $a \in \bar{B}$ iff $a\delta B$ this condition can be written

$$a \in \bar{B} \,\&\, b \in \bar{C} (b \in B) \Rightarrow a \in \bar{C},$$

or equivalently

$$a \in \bar{B} \,\&\, B \subseteq \bar{C} \Rightarrow a \in \bar{C},$$

and this is clearly true because $B \subseteq \bar{C}$ implies $\bar{B} \subseteq \bar{C}$.

So far we have established the necessity of conditions (P.4) and (P.5a). Let us now prove their sufficiency:

By (P.5a) $x\delta A_\delta$ and $y\delta A$ $(y \in A_\delta)$ together imply $x\delta A$, and this in turn yields $x \in A_\delta$. The implication

$$x\delta A_\delta \Rightarrow x \in A_\delta$$

just established proves $A_\delta$ to be closed. We infer from (P.4) that $A \subseteq A_\delta$, and $A \subseteq A_\delta \subseteq \bar{A}$ now yields $A_\delta = \bar{A}$. ∎

It is not hard to see that each of the four conditions underlying Proposition 2. can be derived from the conditions which serve to define LE-proximities. (See [3], (19.1) Definition.) Indeed, we have the implications

$$\text{(i)} \Rightarrow \text{(R2)}; \quad \text{(ii)} \Rightarrow \text{(P3)};$$

$$\text{(iv)} \Rightarrow \text{(P.4)}; \quad \text{(iii)} \Rightarrow \text{(P.5a)}.$$

Thus the statement of our Proposition 2. will in particular be valid if $\delta$ is an LE-proximity. Seen the fact that LODATO's LO-proximities differ from LEADER's LE-proximities only by the imposition of an additional (commutativity) condition, the statement of Proposition 2. is, of course, valid for LO-proximities too.

## II

Without explicitely saying so, Theorem (1.18) of [2] describes the notion of symmetric generalized proximity space within the frame of CSÁSZÁR's syntopogeneous theory. As a matter of fact, this theorem essentially says that a symmetric generalized proximity space can be regarded as a symmetric topogeneous order satisfying the additional condition[1])

(Q.6) $A < B$ implies that, for all $C$, $A < C$ or there exists $x \in X - C$ with $x < B$,

providing thereby a counterpart, for symmetric generalized proximity spaces, of (7.26) in [1].

In section II. of [2], for a non-void subset $\mathscr{U}$ of $\mathbf{P}(X \times X)$ the following axioms are considered:

(M.1) For every $U \in \mathscr{U}$, $U \supseteq \Delta$.

(M.2) $\bigcap \{U \mid U \in \mathscr{U}\} = \Delta$.

(M.3) For every $U \in \mathscr{U}$, $U = U^{-1}$.

[1]) $A < B$ iff $A\bar{\delta}(X-B)$. In [2], $A \ll B$ is written instead of $A < B$.

(M.4) For every $A\in\mathbf{P}(X)$ and $U$, $V$ in $\mathcal{U}$ there is a $W\in\mathcal{U}$ such that $W[A]\subseteq$ $\subseteq U[A]\cap V[A]$.

(M.5) For every $U$, $V$ in $\mathcal{U}$, $U\cap V\in\mathcal{U}$.

(M.6) For every $A$, $B$ in $\mathbf{P}(X)$ and $U\in\mathcal{U}$, if $V[A]\cap B\neq\emptyset$ for all $V\in\mathcal{U}$, then there exists $x\in B$ and there exists a $W\in\mathcal{U}$ such that $W[x]\subseteq U[A]$.

(M.7) For every $U\in\mathcal{U}$ there exists a $V\in\mathcal{U}$ such that $V\circ V\subseteq\mathcal{U}$.

(M.8) If $U\in\mathcal{U}$ and $V\subseteq X\times X$ and $U\subseteq V$ and $V=V^{-1}$, then $V\in\mathcal{U}$;
and the following definitions are adopted:

$\mathcal{U}$ is a *symmetric generalized uniformity* if it satisfies conditions (M.1, 3, 4, 6, 8);

$\mathcal{U}$ is a *correct uniformity* if it satisfies conditions (M.1, 3, 4, 7, 8);

$\mathcal{U}$ is a *symmetric uniformity* if it satisfies conditions (M.1, 3, 5, 7, 8).

It is our aim to give syntopogeneous descriptions of these three notions, providing thereby counterparts to (7.31)—(7.33) in [1].

Let us begin with symmetric uniformities: their definition is closely akin to, but more restrictive than CsÁszÁr's definition of a uniform structure[2]). A comparison of the two definitions yields the following table of correspondence[3]):

$$(\mathrm{U}_1) = (\mathrm{M.1})$$
$$(\mathrm{U}_2) \Leftarrow (\mathrm{M.5})$$
$$(\mathrm{U}_3) = (\mathrm{M.7})$$
$$(\mathrm{U}_4) = (\mathrm{M.3})$$
$$(\mathrm{M.8})$$

Accordingly, CsÁszÁr's result (7.31)—(7.33) remains valid for Mozzochi's symmetric uniformities, and in order to obtain a syntopogeneous characterization of symmetric uniformities, it will be sufficient to account for the surplus conditions in their definition.

Now CsÁszÁr's result just mentioned can be formulated as follows[4]):

**Theorem.** ([1], Chapter 7.) *Uniformities and symmetric syntopologies on a given set $X\neq\emptyset$ can be identified in the following sense:*

(1) *If $\mathcal{U}$ is a uniformity on $X$, then putting*

$$A <_U B \quad \textit{iff} \quad \left.\begin{matrix} x\in A \\ (x,y)\in U\end{matrix}\right\} \Rightarrow y\in B$$

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[2]) See [1], pp. 65—67, and in particular the footnote on p. 67.

[3]) $(U_2)$: $U', U''\in\mathcal{U}\Rightarrow(\exists\, U\in\mathcal{U})U\subseteq U'\cap U''$.

[4]) Whenever in the sequel we are going to speak about Császár's Theorem or simply about the Theorem, it will be this result that we shall mean.

*for each $U \in \mathcal{U}$ we obtain a symmetric syntopology*

$$\mathcal{S}_{\mathcal{U}} = \{<_U | U \in \mathcal{U}\} \quad on \quad X.$$

(2) *If $\mathcal{S}$ is a symmetric syntopology on $X$, then putting*

$$(x, y) \in U_< \quad iff \quad x \not< X - y$$

*for each $< \in \mathcal{S}$, we obtain a uniformity*

$$\mathcal{U}_{\mathcal{S}} = \{U_< | < \in \mathcal{S}\} \quad on \quad X.$$

(3) *The mappings*

$$\mathcal{U} \to \mathcal{S}_{\mathcal{U}} \quad and \quad \mathcal{S} \to \mathcal{U}_{\mathcal{S}}$$

*are one-to-one correspondences, inverse to each other, between the sets of all uniformities and all symmetric syntopologies on X.* ∎

*Remark.* As is known (see [1], (5.39)—(5.43)), the formulae in parts (1) and (2) of this theorem establish a natural one-to-one correspondence between biperfect topogeneous orders $<$ on $X$ and reflexive subsets of $X \times X$, and in particular between symmetric perfect topogeneous orders on $X$ and reflexive, symmetric subsets of $X \times X$. This one-to-one correspondence will be relied upon in the proof of the following propositions. ∎

The desired syntopogenous characterization of symmetric uniformities is now given by the following

Proposition 3. *A uniformity is a symmetric uniformity in Mozzochi's sense, iff the corresponding symmetric syntopology satisfies the following condition*[5]*:*

$$\text{(D)} \qquad \left.\begin{matrix} < \in \mathcal{S} \\ <_1 \subseteqq < \end{matrix}\right\} \Rightarrow <_1 \in \mathcal{S}.$$

PROOF. (i) The condition is necessary:

Suppose $\mathcal{U}$ is a symmetric uniformity, $\mathcal{S}_{\mathcal{U}} = \mathcal{S}$ the corresponding symmetric syntopology, $< \in \mathcal{S}$, an $<_1$ a symmetric perfect topogeneous order satisfying $<_1 \subseteqq <$. Then by

$$x \not< X - y \Rightarrow x \not<_1 X - y$$

we have $U_< \subseteqq U_{<_1}$. Now by (M.8) $U_{<_1} \in \mathcal{U}$, and so $<(U_{<_1}) = <_1 \in \mathcal{S}$. This establishes (D).

(ii) The condition is sufficient:

Let $\mathcal{S}$ be a symmetric syntopology satisfying (D). We need only to show that

$$\mathcal{U}_{\mathcal{S}} = \{U_< | < \in \mathcal{S}\}$$

satisfies (M.5) and (M.8).

[5]) In formulating the descending condition (D), we suppose all occurring relations „$<$" to be symmetric perfect topogenous orders. Thus (D) simply says that if a symmetric perfect topogenous order belongs to $\mathcal{S}$, then any smaller symmetric perfect topogenous order also belongs to $\mathcal{S}$.

First of all we remark that on the basis of (D) and of condition $(S_1)$ in the definition of a syntopogeneous structure, we have the implication

$$\text{(SP)} \qquad <, <_1 \in \mathcal{S} \Rightarrow (< \cup <_1)^{(sp)} \in \mathcal{S}$$

where $(< \cup <_1)^{(sp)}$ denotes the smallest symmetric perfect topogeneous order containing $< \cup <_1$. (The existence of such a one is assured, because $\subseteq$ is both symmetric and perfect, and symmetry as well as perfectness are preserved when forming arbitrary intersections.)

As to (M.5), if $U, V \in \mathcal{U}_{\mathcal{S}}$, i.e. $U = U_{<_1}$, $V = U_{<_2}$ $(<_1, <_2 \in \mathcal{S})$, then by (SP) $< = (<_1 \cup <_2)^{(sp)} \in \mathcal{S}$, and $U \cap V = U_< \in \mathcal{U}_{\mathcal{S}}$.

Here the validity of $U \cap V = U_<$ follows from the fact that the correspondence $< \to U_<$ is order-inverting, and so the smallest symmetric perfect topogeneous order $<$ containing both $<_1$ and $<_2$ yields the largest reflexive and symmetric relation $U = U_<$ contained in both $U$ and $V$, i.e. $U \cap V$.

In order to establish (M.8), let $U \in \mathcal{U}_{\mathcal{S}}$ i.e. $U = U_<$ $(< \in \mathcal{S})$, and $V \supseteq U$, for some symmetric subset $V$ of $X \times X$.

Then $V = U_{<_1}$ where $<_1 = <_V$ is defined as in part (1) of CsÁszÁr's theorem. Now $V \supseteq U$ implies $<_1 \subseteq <$ and so by (D) $<_1 \in \mathcal{S}$, and consequently $V = U_{<_1} \in \mathcal{U}_{\mathcal{S}}$. ∎

Our next aim is to give a syntopogeneous characterization of correct uniformities.

As we have seen, correct uniformities can be obtained by replacing in the definition of symmetric uniformities condition (M.5) by the weaker condition (M.4): For $U, V \in \mathcal{U}$ and for $A \subseteq X$ given, there is

$$W = W(U, V, A) \in \mathcal{U} \quad \text{so that} \quad W[A] \subseteq U[A] \cap V[A].$$

Let now $U$ be a correct uniformity, and let us consider the set

$$\mathcal{S}_{\mathcal{U}} = \{<_U | U \in \mathcal{U}\}.$$

Conditions (M.1) and (M.3) being valid for $\mathcal{U}$, the elements of $\mathcal{S}_{\mathcal{U}}$ are symmetric perfect topogeneous orders. Also, condition $(S_2)$ in the definition of a syntopogenous structure[6]) remains valid for $\mathcal{S}_{\mathcal{U}}$, because it is implied by (and, in fact, equivalent to) (M.7) valid for $\mathcal{U}$. Moreover, on the basis of condition (M.8), the descending condition (D) is also true for $\mathcal{S}_{\mathcal{U}}$.

In proving Proposition 3 we found that the symmetric syntopology corresponding to a symmetric uniformity satisfies (SP), a condition stronger than $(S_1)$. It will be of some interest to note that this condition (SP) is the exact equivalent, the "syntopogeneous translation" as it were, of condition (M.5). More formally, this equivalence is expressed by the following

**Lemma 1.** *If* $\mathcal{U} = \mathcal{U}_{\mathcal{S}}$ *and* $\mathcal{S} = \mathcal{S}_{\mathcal{U}}$ *correspond to each other in the sense of the Theorem, where* $\mathcal{U}$ *is supposed to satisfy only conditions* (M.1, 3, 7, 8), *then* $\mathcal{U}$ *will satisfy condition* (M.5), *if and only if* $\mathcal{S}$ *satisfies condition* (SP).

PROOF. We have shown in proving Proposition 3. that (SP) valid for $\mathcal{S}$ implies the validity of (M.5) for $\mathcal{U} = \mathcal{U}_{\mathcal{S}}$.

[6]) See [1], (7.1).

Conversely, suppose that $\mathscr{U}$ satisfies condition (M.5).

Now let two symmetric perfect topogenous orders belonging to $\mathscr{S}$, $<_1 = <_{U_1}$ and $<_2 = <_{U_2}$ $(U_1, U_2 \in \mathscr{U})$ be given. By (M.5) $U_1, U_2 \in \mathscr{U} \Rightarrow U_1 \cap U_2 \in \mathscr{U}$, and so $< = <_{U_1 \cap U_2} \in \mathscr{S}$.

Now $<$ so defined is a symmetric perfect topogenous order, and in view of

$$<_U \subseteqq <_V \quad \text{iff} \quad V \subseteqq U$$

it is even the smallest such order containing both $<_1$ and $<_2$. This proves (SP). ∎

We must now find out what will be the weaker condition replacing (SP) (and possibly even $(S_1)$), if instead of (M.5) we have only (M.4) to rely upon. The answer to this question is given by the following

**Lemma 2.** *If $\mathscr{U} = \mathscr{U}_{\mathscr{S}}$ and $\mathscr{S} = \mathscr{S}_{\mathscr{U}}$ correspond to each other in the sense of the Theorem, where $\mathscr{U}$ is supposed to satisfy only conditions* (M.1, 3, 7, 8), *then $\mathscr{U}$ will satisfy condition* (M.4) *if and only if $\mathscr{S}$ satisfies the following condition*

$(S_{1a})$ *For $<_1, <_2 \in \mathscr{S}$ and for $A \subseteqq X$ given, there is $< \in \mathscr{S}$ so that*[7])

$$\left.\begin{matrix} A <_1 B \\ A <_2 C \end{matrix}\right\} \Rightarrow A < B \cap C.$$

Proof. Let condition $(S_{1a})$ be valid for $\mathscr{S}$. Also, let $U, V \in \mathscr{U} = \mathscr{U}_{\mathscr{S}}$ and $A \subseteqq X$. Consider $<_U \in \mathscr{S}$ and $<_V \in \mathscr{S}$. One sees that the definition of $<_U$ by which

$$A <_U B \quad \text{iff} \quad \left.\begin{matrix} x \in A \\ (x, y) \in U \end{matrix}\right\} \Rightarrow y \in B,$$

can be rewritten so:

$$A <_U B \quad \text{iff} \quad U[A] \subseteqq B.$$

Thus we have

$$A <_U U[A] \quad \text{and} \quad A <_V V[A],$$

and by $(S_{1a})$ there exists $< \in \mathscr{S}$ so that

$$A < U[A] \cap V[A].$$

Now $< = <_W$ for some $W \in \mathscr{U}$, and so

$$W[A] \subseteqq U[A] \cap V[A].$$

Suppose now that $\mathscr{U}$ satisfies (M.4). Let $U, V \in \mathscr{U}$, end hence $<_U, <_V \in \mathscr{S} = \mathscr{S}_{\mathscr{U}}$. Let now be

$$A <_U B \quad \text{and} \quad A <_V C,$$

or equivalently

$$U[A] \subseteqq B \quad \text{and} \quad V[A] \subseteqq C.$$

We infer that

$$U[A] \cap V[A] \subseteqq B \cap C.$$

[7]) Of course, this relation $<$ will depend not only on $<_1$ and $<_2$, but also on the set $A$.

Now, by (M.4), for some $W \in \mathscr{U}$ one has

$$W[A] \subseteq U[A] \cap V[A] \subseteq B \cap C,$$

i.e.

$$A < B \cap C \quad \text{for} \quad < \, = \, <_W . \ \blacksquare$$

Now we are able to formulate the following

Proposition 4.

(1) *If $\mathscr{U}$ is a correct uniformity on a set X, then*

$$\mathscr{S}_{\mathscr{U}} = \{<_U \mid U \in \mathscr{U}\}$$

*is a set of symmetric perfect topogeneous orders, satisfying conditions* $(S_{1a})$, $(S_2)$ *and* (D).

(2) *If $\mathscr{S}$ is a set of symmetric perfect topogeneous orders, satisfying conditions* $(S_{1a})$, $(S_2)$ *and* (D), *then*

$$\mathscr{U}_{\mathscr{S}} = \{U_< \mid < \in \mathscr{S}\}$$

*is a correct uniformity.*

(3) *The mappings*

$$\mathscr{U} \to \mathscr{S}_{\mathscr{U}} \quad \text{and} \quad \mathscr{S} \to \mathscr{U}_{\mathscr{S}}$$

*are one-to-one correspondences, inverse to each other, between the sets of all correct uniformities and all systems of symmetric perfect topogeneous orders satisfying conditions* $(S_{1a})$, $(S_2)$ *and* (D) *on X.*

PROOF. (1) Our previous considerations contain a full proof of this first part.

(2) The proof of part (1) can be reversed so as to yield a proof of this second statement. In particular, condition $(S_{1a})$ being but a "translation" of (M.4), we can retranslate $(S_{1a})$ thereby getting back (M.4).

(3) A straightforward consequence of the formulae by which the transition from $\mathscr{U}$ to $\mathscr{S}_{\mathscr{U}}$ and from $\mathscr{S}$ to $\mathscr{U}_{\mathscr{S}}$ is being realized[8]. ■

Finally, let us try and find a syntopogenous characterization for symmetric generalized uniformities. We arrive at this notion by replacing condition (M.7) in the definition of correct uniformities by (M.6):

For every $A, B \subseteq X$ and $U \in \mathscr{U}$, if $V[A] \cap B \neq \emptyset$ for all $V \in \mathscr{U}$, then there exists $x \in B$ and there exists a $W \in \mathscr{U}$ such[9]) that $W[x] \subseteq U[A]$. ■

It is not hard to find the syntopogenous counterpart of this condition:

$(S'_{2a})$ For every $A, B \subseteq X$ and $U \in \mathscr{U}$, if $A <_V H$ implies $B \cap H \neq \emptyset$ for all $V \in \mathscr{U}$, then there exists $x \in B$ and there exists $W \in \mathscr{U}$ such that

$$A <_U H \quad \text{implies} \quad x <_W H. \ \blacksquare$$

In formulating this condition, we have used the representation $\mathscr{S} = \mathscr{S}_{\mathscr{U}}$ of $\mathscr{S}$. A formulation referring to $\mathscr{S}$ only and leaving without mention the corresponding

[8]) The transition formulae being the same as in the Theorem, the proof too will be the same as that of part (3) there.

[9]) Of course, $x = x(A, U, B)$ and $W = W(A, B, U, X)$.

$\mathcal{U}$ is also easy to obtain. With a self-explanatory change in notations, we indeed arrive at the following equivalent formulation:

($S_{2a}$) For every $A, B \subseteq X$ and $< \in \mathcal{S}$, if $A <' H$ implies $B \cap H \neq \emptyset$ for all $<' \in \mathcal{S}$, then there exists $x \in B$ and there exists $<_1 \in \mathcal{S}$ such that

$$A < H \quad \text{implies} \quad x <_1 H. \blacksquare$$

We have denoted this condition by ($S_{2a}$) because it is weaker than ($S_2$). The proof of this can be obtained by giving a "syntopogenous translation" of the reasoning by which Mozzochi proved that (M.7) implies (M.6). (See [2], Remark (2.1).)

Now we see that in order to obtain a syntopogenous characterization of symmetric generalized uniformities, we have only to replace in Proposition 4. as well as in the proof of that proposition, condition ($S_2$) corresponding to (M.7) by condition ($S_{2a}$), the counterpart of (M.6), obtaining thereby the following

Proposition 5. *The syntopogenous counterparts of symmetric generalized uniformities $\mathcal{U}$ are sets $\mathcal{S}$ of symmetric perfect topogenous orders satisfying conditions* ($S_{1a}$), ($S_{2a}$) *and* (D). *The correspondences $\mathcal{U} \to \mathcal{S}_{\mathcal{U}}$ and $\mathcal{S} \to \mathcal{U}_{\mathcal{S}}$ are defined in the same manner as in the foregoing proposition.* $\blacksquare$

## References

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